Q1. $\int_0^3 \sqrt{9-x^2}\,dx$

recognise this is the area of a quater of a circle with radius 3

$$\therefore \int_0^3 \sqrt{9-x^2}\,dx = \frac{1}{4}\times\pi\times 3^2 = \frac{9}{4}\pi$$

Q2.

$$2\iiint_V dV = 2\int_0^{2\pi}\int_0^{1}\int_0^{\pi} d\phi\, dr\, d\theta$$

$$= 2\int_0^{2\pi} \pi\, d\theta = 2\times 2\pi^2$$

$$= 4\pi^2$$

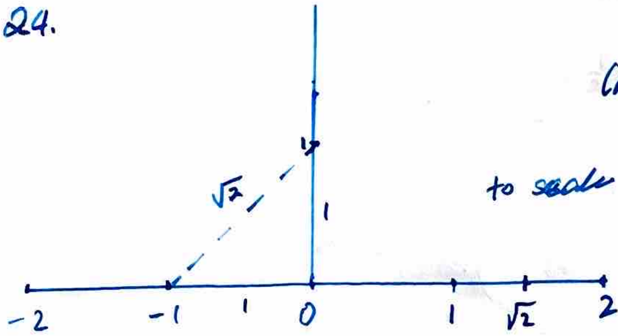
Q3. $\int \frac{\cos x}{3+2\cos x}\,dx$

$$= \frac{1}{2}\int dx - \frac{3}{2}\int \frac{1}{3+2\cos x}\,dx$$

$$= \frac{1}{2}x - 2\times\frac{3}{2}\int \frac{\frac{1}{2}\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+5}\,dx$$

$$= \frac{1}{2}x - \frac{3\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}} + c$$

Q4.



(doesn't count i don't have a compass)

to scale 2cm = 1 unit

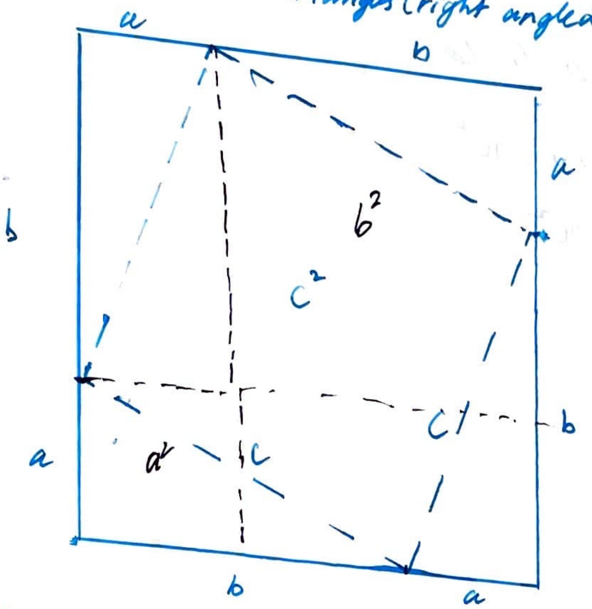
get a compass set it to the dotted length then start at zero

Q5. $\Gamma(\frac{5}{2}) = \frac{\sqrt{\pi}\,(3)!!}{2^2}$

$= \frac{3\sqrt{\pi}}{4}$

Q6

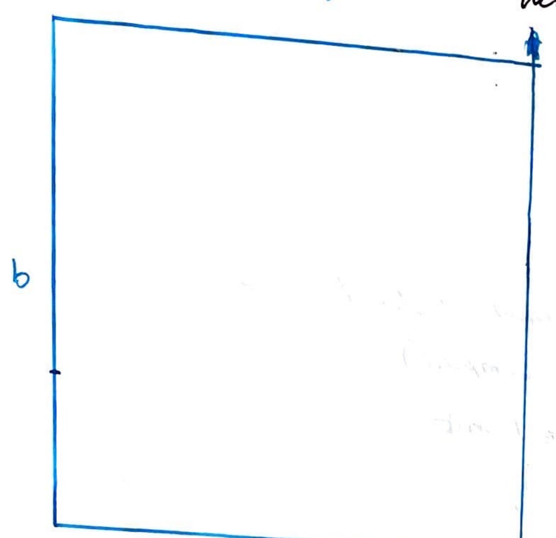
Construct 4 triangles (right angled) with sides a, b, c



$\therefore a^2 + b^2 = c^2$

Push the triangles together

decided to do in black
cus ceebs



Q7 $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$

$\sin(x+h) = \sin(x)\cos(h) + \cos(x) + \sin(h)$ ~~-sinx~~ (can prove using euler's identity)

$= \lim_{h\to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$

$= \lim_{h\to 0} \frac{\sin(x)(\cos(h) - 1)}{h} + \lim_{h\to 0} \frac{\sin(h)\cos(x)}{h}$

$= \sin x \lim_{h \to 0} \left(\frac{\cos(h) - 1}{h} \right) + \cos(x) \lim_{h \to 0} \frac{\sin(h)}{h}$

$= \sin x \times 0 + \cos(x)$ (using L'Hopitals or $\sin(x) = x$ for small x)

Q 8 a)

0, 1, 1, 2, 3, 5, 8, 13, 21, 34

b) the tendrils from each antenna form the fibonacci sequence. You start from the main cardioid and keep going to the next biggest cardioid.

Q9. The answer is 0.

$\int_{\infty}^{\infty} f(x)dx = 0$

however,

$\int_{-\infty}^{\infty} e^{-x^2} dx$

let's start with the probability density function of a normal

$PDF = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

set $\mu = 0$, $\sigma = \frac{1}{\sqrt{2}}$

the integral over the support of a PDF = 1

$\therefore 1 = \int_{-\infty}^{\infty} \frac{1}{\frac{1}{\sqrt{2}} \times \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x^2}{\frac{1}{2}}\right)} dx$

$1 = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} e^{-x^2} dx$

$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$

Q10.

$$\tan^{-1}(x) = \int_0^x \frac{1}{1+t^2}\,dt$$

$$\frac{1}{1+t^2} = 1 - t^2 + t^4 - t^6 + \dots \quad \text{(taylor series expansion)}$$

$$\tan^{-1}(x) = \int_0^x 1 - t^2 + t^4 - t^6 \dots \,dt$$

$$= \frac{t}{1} - \frac{t^3}{3} + \frac{t^5}{5} - \frac{t^7}{7} \dots \Big|_0^x$$

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

$x = 1$

$$\tan^{-1}(1) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots$$

$$= \frac{\pi}{4}$$

Q11.

Calculus is for the rate of change. Analysis has so many forms idtk. Real analysis is calc with proof. other analysis ~~can~~ can be complex.

Q12. easy to see $x=5$ is a soln for both

$(5, 0)$

$f'(x) = 4x^3 - 15x^2$

$f'(5) = 4 \times 5^3 - 15 \times 5^2 = 125$

$g'(x) = 8$

$y = 125(x-5)$

$$\cos(\theta) = \frac{\langle 5, 125 \rangle \cdot \langle 5, 8 \rangle}{\sqrt{5^2+125^2} * \sqrt{5^2+8^2}}$$

$\vec{a} = \langle 5, 125 \rangle$

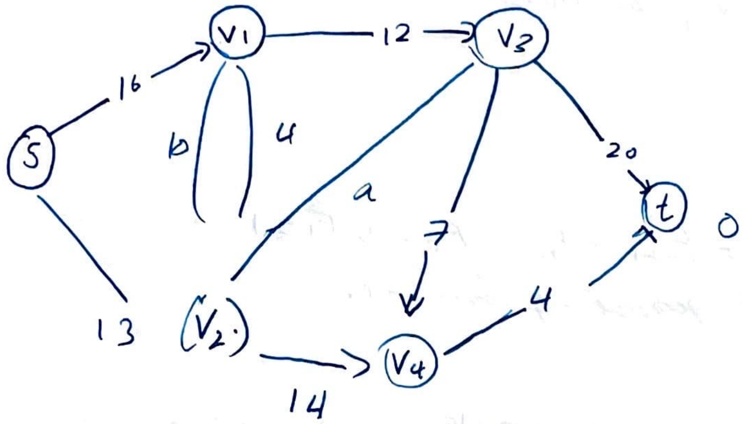
$\vec{b} = \langle 5, 8 \rangle$

~~$\theta \doteq 0.3169$ radians~~

$$= \frac{1125}{1208.568} \approx 0.9464$$

$\theta = 0.08437$ radians

Q13



$$18 + 13 = 31$$

$$S \rightarrow V_2 \rightarrow V_a \rightarrow t$$

Q14.

$$\frac{e^{iz} - e^{-iz}}{2i} = 2$$

$$e^{2iz} - 4i e^{iz} - 1 = 0$$

$$e^{iz} = 2i \pm \sqrt{-3}$$

$$= (2 \pm \sqrt{3}) i$$

$$e^{-y} e^{ix} = (2 \pm \sqrt{3}) i$$

equating real

$$e^{-y} = 2 \pm \sqrt{3}$$

$$e^{ix} = i = e^{-i\frac{\pi}{2}}$$

$$z = \left(\frac{\pi}{2} + 2k\pi\right) + i \ln(2 \pm \sqrt{3})$$

Q15. We will utilise generating functions

$$F(z) = \sum_{n\geq 0} F_n z^n$$

recurrence is $F_{n+2} = F_{n+1} + F_n$, $F_0 = 0, F_1 = 1$
Applying properties of generating functions.

$$\frac{F(z) - F_0 - F_1 z}{z^2} = \frac{F(z) - F_0}{z} + F(z)$$

Par frac

$$F(z) = \frac{z}{1-z-z^2} = \frac{1}{\sqrt{5}}\left(\frac{1}{1-\varphi z} - \frac{1}{1-\bar{\varphi} z}\right)$$

roots of $x^2 - x - 1$ are denoted as φ and $\bar{\varphi} = \frac{1}{2}(1-\sqrt{5})$

$$F_n = \frac{1}{\sqrt{5}}\left(\varphi^n - (\bar{\varphi})^n\right)$$

b) $T(27) = \frac{1}{\sqrt{5}}\left(\left(\frac{1}{2}(1+\sqrt{5})\right)^{27} - \left(\frac{1-\sqrt{5}}{2}\right)^{27}\right)$

$= 196418$

Q16

~~$r^2 + 2r + 1$~~

~~$r = -1$~~

$y_h = $ ~~...~~

Q17. $\sin x \cos x = \frac{1}{2}\sin 2(x)$

areas are $=$

$\therefore \langle \sin(x) \mid \cos(x) \rangle = 0$

Inspect characteristic eq or solve (aka integrating factor)

$y = e^{-x} z$

$y' = e^{-x}(z' - z)$, $y'' = e^{-x}(z'' - 2z' + z)$

~~applying similar~~ applying similar idea of itos lemma

$$e^{-x} z'' = e^{-x}\cos x$$

$$z = -\cos x + c_1 x + c_2$$

$$y = (c_1 x + c_2)e^{-x} - e^{-x}\cos x$$

Q18. dissassembled cube

$$8! \times 12! \times 2^{12} \times 3^{8}$$

Assembled cube

$$\frac{8! \times 12! \times 2^{12} \times 3^{8}}{3 \times 2 \times 2}$$

Q19. Code written in R

Cipher of 3

Rome was not built in a day

Rome?

Q20.

$y = x^2$

$$L = \int_0^4 \sqrt{1 + 4x^2}\, dx$$

$$= \frac{1}{4}\left(2x\sqrt{1+4x^2} + \ln\left|2x + \sqrt{1+4x^2}\right|\right)\Big|_0^4$$

$$= 2\sqrt{65} + \frac{1}{4}\ln\left(8 + \sqrt{65}\right)$$

Q21.

Everyone who is ~~maj~~ majoring in math does not have a friend who needs help with his or her HW.

Q22.

Not rieman integrable, construct a new measure μ (lebesgue measure)

μ(rational numbers) $= 0$, μ(irrational numbers) $= 1$

$$\int_0^1 D(x)\, d\mu = 0 \times 0 + 1 \times 1 = 1$$