Year 9 Mathematics | Topic 1 | Algebra Revision

PEN Education

January 1, 2024

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Contents

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1 Introduction

What does the word *algebra* mean?

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Where does the word **algebra** come from?

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. . . .
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What is an example of *algebra*?

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What can I do with ALGEBRA?

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. . . .

2 Substitution

What is *substitution*?

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. . . .

Definition 1

Pronumeral:

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. . . .
. . . .

Definition 2

Numerical Value:

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. . . .
. . . .

2.1 Examples

1. E valuate $2x$ when $x = 3$ 1

   ....

2. E valuate $5a + 2b$ when $a = 2$ and $b = -3$ 1

   ....

3. E valuate $2p(3q - 2)$ when $p = 1$ and $q = -2$ 1

   ....

4. E valuate $7m - 4n$ when $m = -3$ and $n = -2$ 1

   ....

5. E valuate $a + 2b - 3c$ when $a = 3, b = -5, c = -2$ 1

   ....

2.2 Exercises

1. E valuate $2x - 3y$ when: 2

   (a) $x = \frac{2}{5}, y = -\frac{1}{4}$

   ....
   ....
   ....
   ....
   ....

   (b) $x = \frac{1}{3}, y = \frac{1}{6}$

   ....
   ....
   ....
   ....
   ....

2. E valuate $p^2 - 2q$ when: 2

   (a) $p = -7, q = 2$

   ....
   ....
   ....
   ....
   ....

   (b) $p = -\frac{1}{3}, q = \frac{5}{6}$

   ....
   ....
   ....
   ....
   ....

3 Like Terms

Why should we group like terms?

....

....

Give an example of grouping like terms.

....

....

3.1 Examples

1. W hich of the following are pairs of like terms? 3

   (a) $3x, 2x$
   (b) $3m, 2c$
   (c) $3x^2, 3x$
   (d) $2x^2y, 3yx^2$
   (e) $2mn, 3nm$
   (f) $5y^2, 6y^2x$

2. S implify each expression if possible: 6

   (a) $4a + 7a =$

   ....

   (b) $3x^2y + 4x^2 - 2x^2y =$

   ....

   (c) $5m + 6n =$

   ....

   (d) $9b + 2c - 3b + 6c =$

   ....

   (e) $3z + 5yx - z - 6xy =$

   ....

   (f) $6x^3 - 4x^2 + 5x^3$

   ....

3.2 Exercises

1. S implify: 2

   (a) $\frac{x}{2} + \frac{x}{3}$

   ....
   ....
   ....
   ....

   (b) $\frac{3x}{4} - \frac{2x}{5}$

   ....
   ....
   ....
   ....

2. W hich of the following are pairs of like terms? 4

   (a) $12m, 5m$
   (b) $-6a, 7b$
   (c) $6ab, -7b$
   (d) $6x^2, -7x^2$

3. S implify each expression by collecting like terms. 3

(a) $8b + 3b$ ....

(b) $6x^2 + 4x^2$ ....

(c) $7f - 3f + 9f$.......... ....

4. F ill in the missing term. 3

   (a) $8mn+$........ $= 12mn$ (b) $6m^2-$........... $= m^2$ (c) $-7a^2b+$......... $= a^2b$

5. S implify by collecting like terms. 6

   (a) $8p + 6 + 3p - 2 =$....................
   (b) $10ab + 11b - 12b + 3ab =$............
   (c) $4p^2 - 3p - 8p - 3p^2 =$................
   (d) $-4x^2 + 3x^2 - 3y - 7y =$.............
   (e) $7x^3 + 6x^2 - 4y^3 - x^2 =$...............
   (f) $-3ab^2 + 4a^2b - 5ab^2 + a^2b =$ ....

6. S implify: 6

   (a) $\frac{c}{6} + \frac{c}{7}$ ....
   (b) $\frac{x}{7} - \frac{x}{8}$ ....
   (c) $c - \frac{c}{7}$ ....
   (d) $\frac{2x}{3} + \frac{x}{4}$ ....
   (e) $\frac{5x}{3} + \frac{x}{2}$ ....
   (f) $\frac{5x}{11} - \frac{2x}{3}$ ....

4 Multiplication and Division

This part is interesting. In the last section we saw that we could **not** further simplify terms that were different such as $2x + 4y$. But now with multiplication and division we can! $2x \times 4y = 8xy$ and $2x \div 4y = \frac{\not{2}x}{\not{4}y} = \frac{x}{2y}$. Isn't that cool?
Okay, now you guys try:

4.1 Examples

1. M ultiplications 6

(a) $4 \times 3a =$ ....

(c) $4m \times 5m =$ ....

(e) $3x \times (-6) =$ ....

(b) $2d \times 5e =$ ....

(d) $3p \times 2pq =$ ....

(f) $-5ab \times -3bc$ ....

2. D ivisions 6

(a) $24x \div 6 =$ ....

(c) $-18x^2 \div (-3)$ ....

(e) $\frac{12x}{21}$ ....

(b) $36a \div 4$ ....

(d) $\frac{15a}{3}$ ....

(f) $\frac{-24xy}{6y}$ ....

4.2 Exercises

1. R ewrite as a single fraction: 6

(a) $\frac{2a}{5} \times \frac{a}{4} =$ ....

(c) $\frac{4p}{q} \times \frac{3}{2p} =$ ....

(e) $\frac{2x}{3} \div \frac{3x}{5} =$ ....

(b) $\frac{3x}{7} \times \frac{5y}{12} =$ ....

(d) $\frac{15}{x} \times \frac{2}{3x} =$ ....

(f) $\frac{6a}{7b} \div \frac{2ab}{3} =$ ....

2. S implify 6

(a) $5c \times 2d =$ ....

(c) $-2m \times (-4m) =$ ....

(e) $7 \times 15p \div 21 =$ ....

(b) $-6l \times (-5m) =$ ....

(d) $24a^2 \div 8 =$ ....

(f) $18y \div 6 \times 2 =$ ....

3. S implify by first cancelling out common factors: 8

(a) $\frac{14p}{21} =$ ....

(b) $\frac{22x^2}{33} =$ ....

(c) $\frac{2xy}{6xy} =$ ....

(d) $\frac{-4xy}{8x} =$ ....

(e) $\frac{2y}{5} \times \frac{y}{4} =$ ....

(f) $\frac{p}{6q} \times \frac{9p}{4q} =$ ....

(g) $\frac{2yz}{5xy} \times \frac{3xy}{4yz} =$ ....

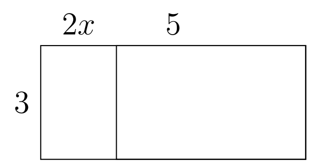
(h) $\frac{2y}{5} \div \frac{y}{4} =$ ....

5 Simple Expansion of Brackets

Often times algebraic concepts have a geometric meaning too. You've now done enough arithmetic with pronumerals to be able to learn this secret of the universe.
Consider $3(2x+5)$. You can expand this by distributing the 3 to each term in the brackets like so:

$$\overset{1\quad 2}{(3)(2x+5)} = 6x + 15 \tag{1}$$

Or you can think about this as having some kind of original rectangle with dimensions 3 by $2x$ and then extending the width by 5.



Now finding the area of the enlarged shape is algebraically equivalent to $3(2x+5)$ and often times expanding this will make substitution easier if you know what the value of x is!
These kinds of expansions are the backbone of mathematics and becoming proficient at these will help you simplify harder problems. Let's get better at expanding:

5.1 Examples

1. Expand: 3

(a) $2(a+3) =$ ....

(b) $3(x-2) =$ ....

(c) $4(2m-7) =$ ....

2. Now try: 3

(a) $5(a+1)+6 =$ ....

(b) $4(2b-1)+7 =$ ....

(c) $6(d+5)-3d =$ ....

3. C an you handle some more terms? [2]

(a) $2(b+5)+3(b+2)=$ ....

(b) $3(x-2)-2(x+1)=$ ....

5.2 Exercises

1. H ave a go at these ones yourselves: [10]

(a) $\frac{3}{5}(6x+\frac{7}{3})=$ ....

(b) $\frac{4}{3}(6x+11)+\frac{2}{3}=$ ....

(c) $-12(4y-5)=$ ....

(d) $\frac{2}{3}(12p+6)=$ ....

(e) $-\frac{1}{2}(10d-6)=$ ....

(f) $-\frac{4}{5}(25m-100)=$ ....

(g) $\frac{3}{5}(\frac{x}{6}+\frac{1}{3})=$ ....

(h) $-\frac{3}{5}(\frac{a}{3}-\frac{2}{3})=$ ....

(i) $c(c-5)=$ ....

(j) $2i(5i+7)=$ ....

6 Binomial Products

Welcome to some respectable mathematics. Binomals look like this: (x+something)(y+something else). We are going to learn how to expand any variant of these, and then we will look at the special cases when x and y are the same and the *something*'s are also the same; i.e. $(x+a)(x+a)$. (There is a quick trick for solving these). Then we shall conclude the class with the second special case of the binomials - *The Difference of Two Squares*. They come in the shape of $(x+a)(x-a)$, and also can be easily expanded with a trick!
Before we get stuck in to the expansion tricks, let's make sure we understand what we are expanding.
What does the prefix ***bi*** mean?

....

....

....

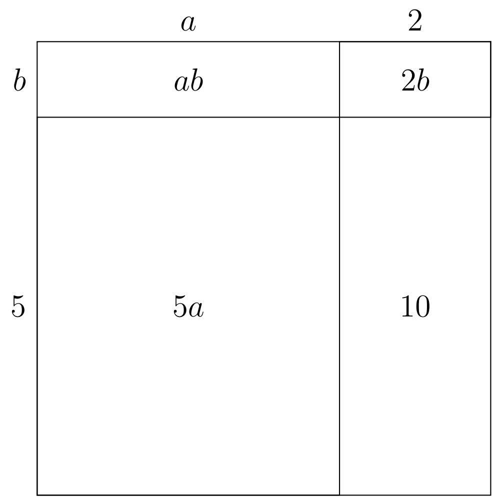
Examples of '**bi**' things include ....

Thus a **bi**nomial means ....

Now let us expand $(a+2)(b+5)$. You just need to distribute each term in the first brackets with every term of the next set of brackets.

$$(a+2)(b+5) = ab+2b+5a+10 \tag{2}$$

If at first you are struggling to remember the steps, just remember the acronym **FOIL**, **F**irst **O**utside **I**nside **L**ast.
Once again this has a geometric interpretation:



And the area can now be computed by adding all the parts: $ab + 2b + 5a + 10$, which is what our algebraic expansion told us too!

6.1 Examples

1. E xpand the following: 4

   (a) $(x+4)(x+5) =$

   ....

   (b) $(x+3)(x-2) =$

   ....

   (c) $(x-4)(x-3) =$

   ....

   (d) $(2y+1)(3y-4) =$

   ....

6.2 Exercises

1. (a) $(a+3)(a+9) =$

   ....
   ....
   ....

   (b) $(a+8)(9+a) =$

   ....
   ....
   ....

   (c) $(p-6)(p+4) =$

   ....
   ....
   ....

   (d) $(x+3)(x-8) =$

   ....
   ....
   ....

   (e) $(x+7)(x-4) =$

   ....
   ....
   ....

   (f) $(5x+1)(x+2) =$

   ....
   ....
   ....

   (g) $(4m+3)(2m-1) =$

   ....
   ....
   ....

   (h) $(2x-7)(3x-1) =$

   ....
   ....
   ....

   (i) $(2b+3)(4b-2) =$

   ....
   ....
   ....

   (j) $(4c+d)(2c-3d) =$

   ....
   ....
   ....

   (k) $(3x-y)(2x+5y) =$

   ....
   ....
   ....

   (l) $(2p-5q)(3q-2p) =$

   ....
   ....
   ....

□

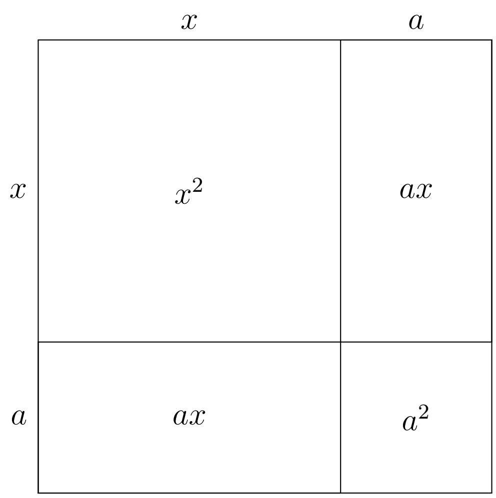
7 Perfect Squares

This is one of the special cases mentioned earlier. Our general binomial looks like $(x+a)(y+b)$, but perfect squares are easier and look like $(x+a)(x+a)$ which can then be simplified to be $(x+a)^2$.
Remember the trick mentioned earlier? This is it:

1. Take the first term, square it

2. Take the last term, square it
3. Multiply all the terms with each other

Thus we have $(x+a)^2 = x^2 + 2ax + a^2$. Simple as that.
Here is the geometric intuition:



We take a square of x units and extend it to a square of length $x+a$:

7.1 Examples

Let's only do a few examples this time. We'll come back and do more practise after covering *differences of two squares.*

1. (a) $(x-5)^2 =$

   ....

   (b) $(x+7)^2 =$

   ....

   (c) $(3x-1)^2 =$

   ....

□

8 Difference of Two Squares

We shall cover this one quickly so you have time to do a brick of exercises after :D. Difference of Two Squares are the second special case of the **binomial** expansion, and come in the form $(x+a)(x-a)$. Expanding this out with our usual **FOIL** method gives $x^2 + \cancel{ax} - \cancel{ax} - a^2$ which just leaves $x^2 - a^2$; how convenient!
This time I will leave the geometric intuition as an exercise, feel free to come to me before next class to explain your ideas!

8.1 Exercises

1. L et's practise: 3

   (a) $(x-5)(x+5) =$

   ....

   ....

   (b) $(3x-4)(3x+4) =$

   ....

   ....

   (c) $(a+b)(a-b) =$

   ....

   ....

2. N ow back to perfect squares: 4

   (a) $(x+1)^2 =$

   ....

   ....

   (b) $(x+5)^2 =$

   ....

   ....

   (c) $(2+x)^2 =$

   ....

   ....

   (d) $(x+20)^2 =$

   ....

   ....

3. T ry a mix now: 6

(a) $(3x-2)(3x+2) =$

. . . .

. . . .

. . . .

(b) $(3a-4b)^2 =$

. . . .

. . . .

(c) $(2x+3y)^2 =$

. . . .

. . . .

(d) $(5a+2b)(5a-2b) =$

. . . .

. . . .

(e) $(\frac{x}{2}+3)^2 =$

. . . .

. . . .

(f) $(3c-b)^2 =$

. . . .

. . . .

9 Homework

1. E valuate $2m(m-3n)$ when: [3]

   (a) $m=3, n=5$ (b) $m=-3, n=-2$ (c) $m=\frac{1}{3}, n=\frac{1}{2}$

2. E valuate $\frac{p+2q}{3r}$ when $p=7, q=-2, r=2$ [1]

3. E valuate $\frac{x+y}{3}$ when $x=-6, y=-5$ [1]

4. F ill in the missing term: [3]

   (a) $2a+$ .... $=7a$ (b) $5m^2-$ .... $=-6m^2n$ (c) $-6lm+$ .... $=lm$

5. S implify by collecting like terms: [6]

   (a) $9a^2+5a^2-12a^2$ (c) $17m^2-14m^2+8m^2$ (e) $7x^3+6x^2-4y^3-x^2$

   (b) $14a^2d-10a^2d-6a^2d$ (d) $-4x^2+3x^2-3y-7y$ (f) $-3ab^2+4a^2b-5ab^2+a^2b$

6. S implify [12]

(a) $4a \times 3b$

(d) $3 \times 12t \div 9$

(g) $\frac{12ab}{4a}$

(j) $\frac{3x}{5} \div \frac{3}{4}$

(b) $-2p \times (-3q)$

(e) $24x \div 8 \times 3$

(h) $\frac{3x}{5} \times \frac{2}{3}$

(k) $\frac{9y}{2} \div 18$

(c) $27y \div 3$

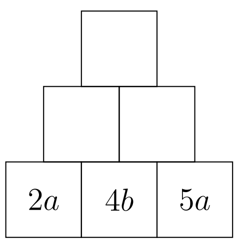
(f) $-\frac{12m}{18}$

(i) $\frac{2}{5a} \times \frac{1}{4a}$

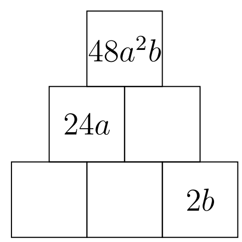
(l) $\frac{5p}{6} \div (-\frac{10p}{3})$

7. Fill in the missing boxes. Each box contains the product of the 2 boxes below it. **4**

(a)



(b)



8. Expand: **8**

(a) $b(b+7)$

(c) $-k(5k-4)$

(e) $4c(2c-d)$

(g) $3p(2-5pq)$

(b) $4h(5h-7)$

(d) $-4x(3x-5)$

(f) $-3x(2x+5y)$

(h) $-10b(3a-7b)$

9. Expand and collect like terms **6**

(a) $\frac{1}{4}(x+2) + \frac{x}{3}$

(b) $\frac{3}{7}(3x+5) + \frac{x}{3}$

(d) $2p(3p+1)-5(p+1)$

. . . .

. . . .

. . . .

(c) $-\frac{1}{2}(3x+2)-\frac{2x}{5}$

(f) $4z(4z-2)-z(z+2)$

. . . .

. . . .

. . . . (e) $2p(3p+1)-4(2p+1)$ . . . .

10. E xpand: $\boxed{8}$

(a) $(x-6)(x-4)$ (c) $(4x+3)(2x-1)$ (e) $(x+3)(x+3)$ (g) $(2x+3)(2x+3)$

. . . .

. . . .

. . . .

(b) $(4x+1)(3x-1)$ (d) $(x-4)(2x+5)$ (f) $(2x-5)(x+3)$ (h) $(\frac{2b}{3}+2)(\frac{b}{5}-2)$

. . . .

. . . .

. . . .

11. F ill in the blanks: $\boxed{6}$

(a) $(x+5)(\_\_\_\_\_\_\_\_\_\_) = x^2+8x+15$

(b) $(x+3)(\_\_\_\_\_\_\_\_\_\_) = x^2-2x-15$

(c) $(3x+4)(\_\_\_\_\_\_\_\_\_\_) = 3x^2+x-4$

(d) $(x+\_\_\_\_\_\_\_\_\_\_)(x+6) = x^2+9x+$ __________

(e) $(2x+3)(\_\_\_\_\_\_\_\_\_\_) = 2x^2+7x+$ __________

(f) $(\_\_\_\_\_\_\_\_\_\_ x-3)(\_\_\_\_\_\_\_\_\_\_ x5 \_\_\_\_\_\_\_\_\_\_)$ $12x^2-x-6$

12. E xpand $\boxed{4}$

(a) $(x-7)^2$ (b) $(a+8)^8$ (c) $(9+x)^2$ (d) $x-11)^2$

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13. E xpand $\boxed{2}$

(a) $(\frac{2x}{5}-1)^2$

. . . .

. . . .

. . . .

(b) $(\frac{3x}{4} + \frac{2}{3})^2$

14. E valuate the following using $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$. [3]

(a) $(1.01)^2$ (b) $(0.99)^2$ (c) $(4.01)^2$

15. E xpand and collect like terms [4]

(a) $(x-2)^2 + (x-4)^2$

(b) $(2x+5)^2 + (2x-5)^2$

(c) $x^2 + (x+1)^2 + (x+2)^2 + (x+3)^2$

(d) $(\frac{x}{2}+1)^2 + (\frac{x}{2}-1)^2$

16. E xpand [6]

(a) $(z-7)(z+7)$

(b) $(10-x)(10+x)$

(c) $(3x-2)(3x+2)$

(d) $(\frac{x}{2}+3)(\frac{x}{2}-3)$

(e) $(\frac{x}{3}+\frac{1}{2})(\frac{x}{3}-\frac{1}{2})$

(f) Is $a^2 - 2a + 1$ a perfect square expansion or a difference of 2 squares?

9.1 Challenge Problems

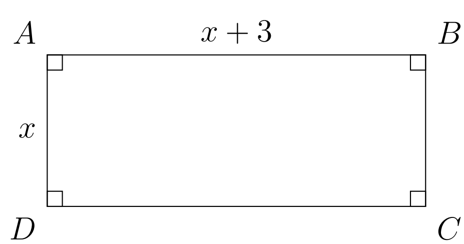
1. (a) Show that the perimeter of the rectangle is $(4x + 6)$cm 5

   (b) Find the perimeter if $AD = 2$cm

   (c) Find x if the perimeter $= 36$cm

   (d) Find the area of $ABCD$ in terms of x

   (e) Find the area of the rectangle if $AB = 6$cm



2. E xpand and collect: 6

   (a) $(x - 1)(x^2 + x + 1)$

   (b) $(x - 1)(x^4 + x^3 + x^2 + x + 1)$

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(c) What do you expect the result of expanding $(x-1)(x^9+x^8+\cdots+1)$ will be?

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. . . .

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Marker's use only.

SECTION	1	2	3	4	5	6	7	8	HW	**Total**
MARKS	/0	/9	/33	/32	/18	/4	/0	/13	/88	/197