Year 9 Mathematics
Topic 3
Consumer Arithmetic [Theory]

PEN Education

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Contents

1 Introduction

In this lesson we will cover the **theory** of *Consumer Arithmetic*.

1. What is consumer arithmetic? (break down the 2 words) [2]

> **Solution:** Consumer means somebody that takes something and uses it. Arithmetic is the manipulation of something by knowing the rules by which it abides.

2. Why is learning about consumer arithmetic important? 1

> **Solution:** By default humans are all consumers, and so being good at the arithmetic of consuming things is something that can be advantageous to all that are human.

3. What will we be doing next lesson and why? 1

> **Solution:** Because consumer arithmetic can become a little confusing we will be doing a *problems workshop* next lesson where we attempt harder questions in a quieter setting. There will also be some questions from prior topics interspersed so that you don't forget everything you have worked so hard to learn.

Here is a box with a number of **buzz words** from today's class. As you come to understand these words, cross them off here.

appreciation GST simple interest compound interest commissions inflation depreciation discounts

2 Review of Percentages

Note: for this particular topic in mathematics, we encourage you to write percentages as decimals rather than fractions. Also, you will be making extensive use of your calculator this lesson and next. Be sure to bring it and use this opportunity to get good at using it. Ask your tutor for help in using this tool as it is powerful and will be a good friend to you for at least the next 3 years!

2.1 Examples

1. Convert the following from percentages to fractions 1

   (a) $42\% =$ 0.42 (b) $12\frac{1}{4}\% =$ $\frac{49}{400}$

2. Convert the following from fractions to percentages 1

   (a) $\frac{3}{5} =$ 60% (b) $\frac{7}{20} =$ 35%

3. The Brilliant Light Bulb Company estimates that 3.5% of its light bulbs are defective. If a shop owner buys 1250 light bulbs to light the shop, how many would he expect to be defective? 1

Solution: Number of defective bulbs $= 1250 \times 3.5\%$

$$= 1250 \times 0.035$$
$$\approx 44 \quad (\text{ Round } 43.75 \text{ to } 44.)$$

4. A typical computer weighs about 25 kg. When it is broken down as waste, it yields about 3 g of arsenic. What percentage of the total is this? 1

Solution: Using grams, the computer weighs 25000 g and the arsenic weighs 3 g. Hence percentage of arsenic $= \frac{3}{25000} \times 100\%$

$$= \frac{3}{250}\%$$

$$= 0.012\%$$

2.2 Exercises

1. Express each percentage as a decimal: 2

   (a) $72\% =$ **0.72**
   (b) $7.6\% =$ **0.076**
   (c) $77\frac{3}{4}\% =$ **0.7775**
   (d) $0.1\% =$ **0.001**

2. Express each percentage as a fraction: 3

   (a) 35% $\frac{7}{20}$
   (b) $33\frac{1}{3}\% =$ $\frac{1}{3}$
   (c) $210\% =$ $\frac{21}{10}$ **or** $2\frac{1}{10}$
   (d) $125\% =$ $\frac{5}{4}$ **or** $1\frac{1}{4}$
   (e) $7.25\% =$ $\frac{29}{400}$
   (f) $112\frac{1}{2}\% =$ $\frac{9}{8}$ **or** $1\frac{1}{8}$

3. Express each fraction or decimal as a percentage: 3

   (a) $\frac{3}{5} =$ **60%**
   (b) $\frac{7}{20} =$ **35%**
   (c) $0.43 =$ **43%**
   (d) $1.2 =$ **120%**
   (e) $0.225 =$ **22.5%**
   (f) $2.03 =$ **203%**

4. Evaluate these amounts, correct to 2 decimal places where necessary. 3

(a) 15% of 40 = **6**

(b) 57% of 1000 = **570**

(c) 120% of 538 = **645.60**

(d) 15.8% of 972 = **153.57**

(e) 2.8% of 318 = **8.90**

(f) 0.1% of 6000 = **6**

5. Find what percentage the first quantity is of the second quantity, correct to 1 decimal place. [3]

   (a) 70 m, 50 m= **140.0%**

   (b) 15 weeks, 60 weeks= **25.0%**

   (c) 60 weeks, 15 weeks= **400.0%**

6. Find what percentage the first quantity is of the second quantity, correct to 2 decimal places where necessary. You will need to express both quantities in the same unit first. [3]

   (a) 68 cents, \$5.00= **13.60%**

   (b) 7 g, 3 kg= **0.23%**

   (c) 15 days, 3 years= $\approx$ **1.37%**

   (d) 4 km, 250 m= **1600.00%**

   (e) 1 year, 1 day= $\approx$ **0.27%**

   (f) 56 cm, 2.4 km= **0.02%**

7. There are 740 students at a primary school, 5% of whom have red hair. Calculate the number of students in the school who have red hair. [2]

   Solution: 37 students

8. A soccer match lasted 92 minutes (including injury time). If team A was in possession for 55% of the match, for how many minutes and seconds was team A in possession? [2]

   Solution: 50 minutes and 36 seconds

3 Using Percentages

Why did we make you just do so many trivial percentage conversions?

Solution: Because in financial mathematics (which is today's topic), percentages are very prominent. They pop up everywhere, from the amount of interest you get on a loan to the rate at which an item depreciates. You must become professionals at working with percentages!

3.1 Examples

1. Joshua saves 12% of his after-tax salary every week. If he saves \$90 a week, what is his after-tax salary? [1]

Solution: Savings = after-tax salary $\times 12\%$

Reversing this:

$$\begin{aligned}\text{after-tax salary} &= \text{savings} \div 12\% \\ &= 90 \div 0.12 \\ &= \$750\end{aligned}$$

2. Sterling silver is an alloy that is made up of 92.5% by mass silver and 7.5% copper. [2]

   (a) How much sterling silver can be made with 5 kg of silver and unlimited supplies of copper?

   Solution: Mass of silver = mass of sterling silver $\times 92.5\%$

   Reversing this:

   mass of sterling silver = mass of silver $\div 92.5\%$

   $$\begin{aligned}&= 5 \div 0.925 \\ &\approx 5.405 \text{ kg}\end{aligned}$$

   (b) How much sterling silver can be made with 5 kg of copper and unlimited supplies of silver?

   Solution: Mass of copper = mass of sterling silver $\times 7.5\%$

   Reversing this:

   mass of sterling silver = mass of copper $\div 7.5\%$

   $$\begin{aligned}&= 5 \div 0.075 \\ &\approx 66.667 \text{ kg}\end{aligned}$$

3. The Eureka Gallery charges a commission of 9.2%. [2]

   (a) The Australian painting Showing the Flag at Bakery Hill was sold recently for \$180000. How much did the Gallery receive, and how much was left for the seller?

Solution: Commission $= 180000 \times 9.2\%$

$$= 180000 \times 0.092$$
$$= \$16560$$

Amount received by seller $= 180000 - 16560$

$$= \$163440$$

(b) The Gallery received a commission of \$7912 for selling the painting Ned at the Glen. What was the selling price of the painting, and what did the seller actually receive?

Solution: Commission = selling price $\times 9.2\%$
Reversing this:
selling price = commission $\div 9.2\%$

$$= 7912 \div 0.092$$
$$= \$86000$$

Amount received by seller $= 86000 - 7912$

$$= \$78088$$

4. The Budget Shoe Shop spent \$6600000 last year buying shoes and paying salaries and other expenses. They made a 2% profit on these costs. 3

(a) What was their profit last year?

Solution: Profit $= 6600000 \times 2\%$

$$= 6600000 \times 0.02$$
$$= \$132000$$

(b) What was the total of their sales?

Solution: Total sales = total costs + profit

$$= 6600000 + 132000$$
$$= \$6732000$$

(c) In the previous year, their costs were \$5225000 and their sales were only \$5145000. What percentage loss did they make on their costs?

Solution: Last year, loss = total costs - total sales

$$= 5225000 - 5145000$$
$$= \$80000$$

Percentage loss $= \left(\frac{80000}{5225000} \times \frac{100}{1}\right)\%$

$$\approx 1.53\%$$

5. Joe's tile shop made a profit of 5.8% on total costs last year. If the actual profit was \$83000, what were the total costs, and what were the total sales? [1]

Solution:

$$\text{Profit} \ = \ \text{costs} \ \times 5.8\%$$

Reversing this, costs = profit $\div 5.8\%$

$$= 83000 \div 0.058$$

$\approx \$1431034$, correct to the nearest dollar.

Hence, total sales = profit + costs

$$\approx 83000 + 1431034$$

$$= \$1514034$$

6. Income tax in the nation of Immutatia is calculated as follows. [4]

- There is no tax on the first \$12000 that a person earns in any one year.
- From \$12001 to \$30000, the tax rate is 15 c for each dollar over \$12000.
- From \$30001 to \$75000, the tax rate is 25 c for each dollar over \$30000.
- Over \$75000, the tax rate is 35 c for each dollar over \$75000.

Find the income tax payable by a person whose taxable income for the year is:

(a) \$10600

Solution: There is no tax on an income of \$10600.

(b) \$25572

Solution: Tax on first \$12000 = \$0
Tax on remaining \$13572 = 13572 × 0.15

$$= \$2035.80$$

This is the total tax payable.

(c) \$62300

Solution: Tax on first \$12000 = \$0
Tax on next \$18000 = 18000 × 0.15

$$= \$2700$$

Tax on remaining \$32300 = 32300 × 0.25

$$= \$8075$$

Total tax = 2700 + 8075

$$= \$10775$$

(d) \$455000

Solution: Tax on first \$12000 = \$0
Tax on next \$18000 = \$2700 (see part c)
Tax on next \$45000 = 45000 × 0.25

$$= \$11250$$

Tax on remaining \$380000 = 380000 × 0.35

$$= \$133000$$

Total tax = 2700 + 11250 + 133000

$$= \$146950$$

3.2 Exercises:

1. Find the quantity, given that: 2

(a) 2% of it is \$12

> **Solution:** Quantity = \$12 / 0.02 = \$600

(c) 30% of it is 36 minutes

> **Solution:** Quantity = 36 minutes / 0.3 = 120 minutes or 2 hours

(b) 6% of it is 750 g

> **Solution:** Quantity = 750 g / 0.06 = 12500 g or 12.5 kg

(d) 90% of it is 54 cm

> **Solution:** Quantity = 54 cm / 0.9 = 60 cm

2. In each part, find the price if: [2]

(a) a deposit of \$360 is 30% of the price

> **Solution:** Price = \$360 / 0.3 = \$1200

(b) a deposit of \$168 is 15% of the price

> **Solution:** Price = \$168 / 0.15 = \$1120

3. Find the original quantity, correct to a suitable number of decimal places, if: [2]

(a) 23% of it is 100 kg

> **Solution:** Quantity = 100 kg / 0.23 = 434.78 kg

(c) 0.92% of it is 1.86 hectares

> **Solution:** Quantity = 1.86 hectares / 0.0092 = 20.22 hectares

(b) 0.2% of it is 4 mm

> **Solution:** Quantity = 4 mm / 0.002 = 2000 mm or 2 m

(d) 97% of it is \$700

> **Solution:** Quantity = \$700 / 0.97 = \$721.65

4. Cameron and Wendy together earn \$1156 per week after tax. Of this, they pay \$460 off their mortgage, \$185 for groceries, and \$260 for their car and transport, and they save \$124. [2]

(a) Express each amount as a percentage of their weekly income, correct to the nearest 1%.

> **Solution:** Mortgage: (\$460/\$1156) $\times$ 100 $\approx$ 39.8% (rounded to 40%)
> Groceries: (\$185/\$1156) $\times$ 100 $\approx$ 16.0% (rounded to 16%)
> Car and transport: (\$260/\$1156) $\times$ 100 $\approx$ 22.5% (rounded to 23%)
> Savings: (\$124/\$1156) $\times$ 100 $\approx$ 10.7% (rounded to 11%)

(b) Find how much is unaccounted for in the list above, and what percentage it is of their weekly income.

Solution: Unaccounted for: $\$1156 - (\$460 + \$185 + \$260 + \$124) = \$1156 - \$1029 = \$127$
Percentage: $(\$127/\$1156) \times 100 \approx 11.0\%$ (rounded to 11%)

4 Simple Interest

There are 2 types of interest, *simple* and *compound*. In this section we will look at **simple** interest and understand what makes it so simple...

One thing that you must realise is that simple interest always applies only to the original amount. For example, if I borrow $1000 from you and you charge me 5% interest on that every year, in one year I will owe you $1050, then the year after that $1100, etc, etc with the amount owed increasing by 5% or $50 a year.

On the other hand, we have compound interest which will apply the interest to however much money is *owed*. Take the above example again. I borrow $1000, you give a rate of 5% interest - but this time you tell me it's **compounded**!

The difference is after the first year I'll owe you $1050, but the year after that I will owe you 5% ontop of $1050! Which comes out to be $1102.5. See how that is more than $1100?

Now whilst it might not seem like much at the start, COMPOUND interest is ***very*** powerful.

> *Compound interest is the eighth wonder of the world. S/he who understands it, earns it, s/he who doesn't, pays it.*
>
> — Albert Einstein

4.1 Examples:

1. Find the simple interest on $8000 for eight years at 9.5% p.a.

Solution:

$$\begin{aligned} I &= PRT \\ &= 8000 \times 9.5\% \times 8 \\ &= 8000 \times 0.095 \times 8 \\ &= \$6080 \end{aligned}$$

2. Jessie borrows \$3000 from her parents to help buy a car. They agree that she should only pay simple interest. Five years later she pays them back \$3600, which includes simple interest on the loan. What was the interest rate?

Solution: The total interest paid was \$600, the principal was \$3000 and the time was 5 years.

$$\begin{aligned} I &= PRT \\ 600 &= 3000 \times R \times 5 \\ 600 &= 15000 \times R \\ R &= \frac{600}{15000} \times 100\% \\ &= 4\% \end{aligned}$$

Simple interest formula

- Suppose that a principal P is invested for T years at an interest rate R p.a. Then the total interest I is given by:

$$I = PRT$$

Remember that R is a percentage. If the interest rate is 5%, then $R = 0.05$.

- If the interest rate R is given per year, the time T must be given in years.
- The formula has four pronumerals. If any three are known, the fourth can be found by substitution.

4.2 Exercises:

1. \$12000 is invested at 7% p.a. simple interest for five years. 2

(a) How much interest will be earned each year?

Solution: Annual interest $= \$12000 \times 7\% = \$840$

(b) Find how much interest will be earned over the five-year period.

Solution: Total interest $= \$840 \times 5 = \$4200$

2. \$2000 is invested at 6.75% p.a. simple interest for three years.

(a) How much interest will be earned each year?

Solution: Annual interest $= \$2000 \times 6.75\% = \$135$

(b) Find how much interest will be earned over the three-year period.

Solution: Total interest $= \$135 \times 3 = \$405$

3. Find the total simple interest earned in each of these investments. 3

(a) \$400 for three years at 6% p.a.

Solution: Total interest $= \$400 \times 6\% \times 3 = \$72$

(b) \$850 for six years at 4.5% p.a.

Solution: Total interest $= \$850 \times 4.5\% \times 6 = \$229.50$

(c) \$15000 for 12 years at 8.4% p.a.

Solution: Total interest $= \$15000 \times 8.4\% \times 12 = \$15120$

4. Find the time T for \$2000 of simple interest on a principal of \$8000 at a rate of 5% p.a. 1

Solution: $2000 = 8000 \times 5\% \times T$ $T = \frac{2000}{8000 \times 5\%} = 5$ years

5. Find the rate R p.a. for \$7200 of simple interest on a principal of \$8000 over 12 years. 1

Solution: $7200 = 8000 \times R \times 12$ $R = \frac{7200}{8000 \times 12} = 0.075$ or 7.5% p.a.

6. Find the principal P for \$3500 of simple interest at a rate of 7% p.a. over 10 years.

Solution: $3500 = P \times 7\% \times 10$ $P = \frac{3500}{7\% \times 10} = \$5000$

7. Regan has arranged to borrow \$10000 at 9.5% p.a. for four years. She will pay simple interest to the bank every year for the loan, with the principal remaining unchanged. How much interest will Regan pay over the four years of the loan? 2

Solution: Annual interest $= \$10000 \times 9.5\% = \$950$
Total interest over 4 years $= \$950 \times 4 = \$3800$

8. Tyler intends to live on the interest on an investment with the bank at 8.6% p.a. simple interest. She will receive \$68000 simple interest every year from the investment. How much money has she invested? 2

Solution: Principal $= \$68000 \ / \ 8.6\%$
Principal $= \$68000 \ / \ 0.086 \approx \$790697.67$

5 Percentage Increase & Decrease

When a quantity is increased or decreased, the change is often expressed as a percentage of the original amount.

5.1 Examples:

1. **Percentage Increase** 1
The number of patients admitted to St Spyridon's Hospital this year suffering from pneumonia is 56% greater than the number admitted for this condition last year. If 245 pneumonia patients were admitted last year, how many were admitted this year?

Solution: This year's total is $100\% + 56\% = 156\%$ of last year's total.

This year's total $= 245 \times 156\%$

$= 245 \times 1.56$

≈ 382 (correct to the nearest whole number)

2. **Inflation** 1
The war-ravaged nation of Zerbai is experiencing inflation of 35% p.a. as a result of

overspending on its navy and air force. Inflation of 35% means that, on average, prices are increasing by 35% every year.

(a) If the price of water is adjusted in line with inflation, what will an annual bill of \$600 become in the next year? [1]

> **Solution:** Next year's prices are $100\% + 35\% = 135\%$ of last year's prices.
> Next year's bill $= 600 \times 1.35$
>
> $= \$810$

(b) What should an annual salary of \$169000 in one year increase to in the following year if it is adjusted to keep pace with inflation? [1]

> **Solution:** Next year's salary $= 169000 \times 1.35$
>
> $= \$228150$

3. **Percentage Decrease** [1]
The company that Yuri Ivanov works for is going through hard times and has decreased all its salaries by 12%. Yuri is attempting to cut every one of his expenses by the same percentage.

(a) His family's weekly grocery bill averages 450 roubles. What should he try to reduce the weekly price of his groceries to? [1]

> **Solution:** Yuri's new salary is $100\% - 12\% = 88\%$ of his original salary.
> New weekly grocery bill $= 450 \times 0.88$

(b) His monthly rental is 18000 roubles. If he moves apartments, what monthly rental should he try to find? [1]

> **Solution:** New monthly rental $= 18000 \times 0.88 = 396$ roubles
>
> $= 15840$ roubles

4. The Wind Energy Company recently announced that this year's profit of \$1400000 constituted a 35% increase on last year's profit. What was last year's profit? [1]

Solution: This year's profit is $100\% + 35\% = 135\%$ of last year's profit.

Hence this year's profit = last year's profit $\times 1.35$

Reversing this, last year's profit = this year's profit $\div 1.35$

$$\begin{aligned} &= 1400000 \div 1.35 \\ &\approx \$1037037, \text{ correct to the nearest dollar} \end{aligned}$$

5. The price of shares in the Fountain Water Company has decreased by 15% over the last month to \$52.70. What was the price a month ago? 1

Solution: The new share price is $100\% - 15\% = 85\%$ of the old share price.

Hence new price = old price $\times 0.85$

Reversing this, old price = new price $\div 0.85$

$$\begin{aligned} &= 52.70 \div 0.85 \\ &= \$62 \end{aligned}$$

6. The current GST rate is 10% of the pre-tax price. 2

(a) If a domestic plumbing job costs \$630 before GST, how much will it cost after adding GST, and how much tax is paid to the Government?

Solution: After-tax price $= 630 \times 1.10$

$$\begin{aligned} &= \$693 \\ \text{Tax} &= 693 - 630 \\ &= \$63 \end{aligned}$$

Note: 10% of \$630 is \$63.

(b) I paid \$70 for petrol recently. What was the price before adding GST, and what tax was paid to the Government?

Solution: Pre-tax price $= 70 \div 1.10$ (Divide by 1.10 to reverse the process.)

$$\begin{aligned} &\approx \$63.64 \\ \text{Tax} &\approx 70 - 63.64 \\ &= \$6.36 \end{aligned}$$

An aside on **GST**. Let's clear the air and make sure everyone understands what this is: it is a **G**oods and **S**ervice **T**ax. This means that whenever you purchase something in Australia, the government receives 10% of the cost of your item. This makes sense since the government makes living for you possible in the first place...Who pays for the police and firemen? This tax is always included in the cost of your item. i.e. if the banana says it costs \$1.10, then it must have really been \$1.00 plus 10c in tax. In other countries you should be careful though because they might not have included the tax in the displayed price!

5.2 Exercises:

1. Traffic on all roads has increased by an average of 8% during the past 12 months. By multiplying by 108% = 1.08, estimate the number of vehicles now on a road given the number of vehicles the road carried a year ago was: 3

   (a) 10000 per day

   Solution: $10000 \times 1.08 = 10800$ vehicles per day

   (b) 80000 per day

   Solution: $80000 \times 1.08 = 86400$ vehicles per day

   (c) 148000 per day

   Solution: $148000 \times 1.08 = 159840$ vehicles per day

2. Prices have increased with inflation by an average of 3.8% since the same time last year. Find today's price for an item that one year ago cost: 4

   (a) \$200

   Solution: $\$200 \times 1.038 = \$207.60$

   (b) \$1.68

   Solution: $\$1.68 \times 1.038 = \$1.74$

   (c) \$345000

   Solution: $\$345000 \times 1.038 = \$358110$

   (d) \$9430

   Solution: $\$9430 \times 1.038 = \$9790.14$

3. A clothing store is offering a 15% discount on all its summer stock. What is the discounted price of an item with original price: 4

(a) \$80

Solution: $\$80 \times 0.85 = \$68$

(c) \$680

Solution: $\$680 \times 0.85 = \$578$

(b) \$48

Solution: $\$48 \times 0.85 = \$40.80$

(d) \$1.60

Solution: $\$1.60 \times 0.85 = \$1.36$

4. A shoe store is offering a 35% discount at its end-of-year sale. Find the original price of an item whose discounted price is: 4

(a) \$1820

Solution: Original Price = $\$1820/0.65 = \$2800$

(c) \$1.56

Solution: Original Price = $\$1.56/0.65 = \$2.40$

(b) \$279.50

Solution: Original Price = $\$279.50/0.65 = \$430$

(d) \$20.80

Solution: Original Price = $\$20.80/0.65 = \$32$

6 Repeated Increase & Decrease

Now we take the methods from the last section and apply them more than once to the same problem:

6.1 Examples:

1. **Repeated Increase:** 4
The population of Abelsburg in the census three years ago was 46430 . In the three years after the census, however, its population has risen by $6.2\%, 8.5\%$ and 13.1%, respectively.

(a) What was its population one year after the census?

Solution: One year after the census, the population was 106.2% of its original value.
Hence population after one year $= 46430 \times 1.062$

$$\approx 49309, \text{ correct to the nearest whole number}$$

(b) What was its population two years after the census?

> **Solution:** Two years afterwards, the population was 108.5% of its value one year afterwards.
> Hence population after two years $= (46430 \times 1.062) \times 1.085$
>
> $$\approx 53500, \text{ correct to the nearest whole number}$$
>
> Note: Do not use the approximation from part **a** to calculate part **b**. Either start the calculation again, or use the unrounded value from part **a**.

(c) What is its population now, three years after the census?

> **Solution:** Three years afterwards, the population was 113.1% of its value two years afterwards.
> Hence population after three years $= (46430 \times 1.062 \times 1.085) \times 1.131$
>
> $$\approx 60508, \text{ correct to the nearest whole number}$$

(d) What was the percentage increase in population over the three years, correct to the nearest 0.1% ?

> **Solution:**
>
> $$\begin{aligned} &= \text{original population } \times 1.062 \times 1.085 \times 1.131 \\ &\approx \text{original population } \times 1.303 \\ &\approx \text{original population } \times 130.3\% \end{aligned}$$
>
> Hence the population has increased over the three years by about 30.3%.
> Note: The percentage increase of 30.3% is significantly larger than the sum of the three percentage increases,
>
> $$6.2\% + 8.5\% + 13.1\% = 27.8\%.$$
>
> Note: The answer to part **d** does not depend on what the original population was.

2. **Repeated Decrease:** 3
Teresa invested \$75000 from her inheritance in a mining company that has not been very successful. In the first year, she lost 55% of the money, and in the second year, she lost 72% of what remained.

(a) How much does she have left after one year?

Solution: One year later, the percentage remaining was $100\% - 55\% = 45\%$.
Hence amount left after one year $= 75000 \times 0.45$

$$= \$33750$$

(b) How much does she have left after two years?

Solution: Two years later, she had $100\% - 72\% = 28\%$ of what she had after one year.
Hence amount left after two years $= 75000 \times 0.45 \times 0.28$

$$= \$9450$$

(c) What percentage of the original inheritance has she lost over the two years?

Solution: Amount left after two years $=$ original investment $\times 0.45 \times 0.28$

$$= \text{original investment } \times 0.126$$
$$= \text{original investment } \times 12.6\%$$

So she has lost $100\% - 12.6\% = 87.4\%$ of her investment over the two years.

3. **Combinations of increases and decreases:** 2
The volume of water in the Welcome Dam has varied considerably over the last three years. During the first year the volume rose by 27%, then it fell 43% during the second year, and it rose 16% in the third year.

(a) What is the percentage increase or decrease over the three years, correct to the nearest 1% ?

Solution: Final volume $=$ original volume $\times 1.27 \times 0.57 \times 1.16$

$$\approx \text{original volume } \times 0.84$$

Since $0.84 < 1$, the volume has decreased. The percentage decrease is about $100\% - 84\% = 16\%$ over the three years.

(b) If there were 366500 megalitres of water in the dam three years ago, how much water is in the dam now, correct to the nearest 500 megalitres?

Solution: Final volume = original volume $\times 1.27 \times 0.57 \times 1.16$

$$= 366500 \times 1.27 \times 0.57 \times 1.16$$
$$\approx 308000 \text{ megalitres, correct to the nearest 500 megalitres.}$$

This time the sum of the percentages is $27\% - 43\% + 16\% = 0\%$, but the volume of water has changed.

6.2 Exercises:

1. Oranges used to cost \$2.80 per kg, but the price has increased by $5\%, 10\%$ and 12% in three successive years. Multiply by $1.05 \times 1.1 \times 1.12$ to find their price now. 1

Solution: New price = $\$2.80 \times 1.05 \times 1.1 \times 1.12 \approx \$3.50$ per kg

2. The dividend per share in the Electron Computer Software Company has risen over the last four years by 10%, 15%, 5% and 12%, respectively. Find the latest dividend received by a shareholder whose dividend four years ago was: 4

(a) \$1000

Solution: Latest dividend = $\$1000 \times 1.1 \times 1.15 \times 1.05 \times 1.12 \approx \$1408.62$

(b) \$1678

Solution: Latest dividend = $\$1678 \times 1.1 \times 1.15 \times 1.05 \times 1.12 \approx \$2363.62$

(c) \$28.46

Solution: Latest dividend = $\$28.46 \times 1.1 \times 1.15 \times 1.05 \times 1.12 \approx \$40.15$

(d) \$512.21

Solution: Latest dividend = $\$512.21 \times 1.1 \times 1.15 \times 1.05 \times 1.12 \approx \$722.15$

3. Rates in Bullimbamba Shire have risen 7% every year for the last seven years. Find the rates now payable by a landowner whose rates seven years ago were: 4

(a) \$1000

Solution: Current rates = $\$1000 \times (1.07)^7 \approx \$1605.78$

(c) \$2566.86

Solution: Current rates = $\$2566.86 \times (1.07)^7 \approx \$4121.98$

(b) \$346

Solution: Current rates = $\$346 \times (1.07)^7 \approx \$556.14$

(d) \$788.27

Solution: Current rates = $\$788.27 \times (1.07)^7 \approx \$1265.76$

4. A tree, whose original foliage was estimated to have a mass of 1500 kg, lost 20% of its foliage in a storm, then lost 15% of what was left in a storm the next day, then lost 40% of what was left in a third storm. Estimate the remaining mass of foliage. 2

Solution: Remaining mass = $1500 \times 0.8 \times 0.85 \times 0.6 \approx 612$ kg

7 Compound Interest

Remember we introduced compound interest back in section 4? Here are some practise questions now.

Recall that: Compound interest applies interest to the *outstanding amount*, unlike SIMPLE INTEREST which applies it to the *principal* amount.

7.1 Examples:

1. Gail has invested \$100000 for six years with the Mountain Bank. The bank pays her interest at the rate of 7.5% p.a., compounded annually. 6

   (a) How much will the investment be worth at the end of one year?

   Solution: Each year the investment is worth 107.5% of its value the previous year.
   Amount at the end of one year $= 100000 \times 1.075$

   $$= \$107500$$

   (b) How much will the investment be worth at the end of two years?

Solution: Amount at the end of two years $= 100000 \times 1.075 \times 1.075$

$$= 100000 \times (1.075)^2$$
$$= \$115562.50$$

(c) How much will the investment be worth at the end of six years?

Solution: Amount at the end of six years $= 100000 \times (1.075)^6$

$$\approx \$154330.15$$

(d) What is the percentage increase on her original investment at the end of six years?

Solution: Final amount = original amount $\times (1.075)^6$

$$\approx \text{ original amount } \times 1.5433$$

So the total increase over six years is about 54.33%.

(e) What is the total interest earned over the six years?

Solution: Total interest $\approx 154330.15 - 100000$

$$= \$54330.15$$

(f) What would the simple interest on the investment have been, assuming the same interest rate of 7.5% p.a.?

Solution: Simple interest $= PRT$

$$= 100000 \times 0.075 \times 6$$
$$= \$45000$$

Note: Compound interest for two or more years is always greater than simple interest for two or more years.

2. **Compound Interest on a loan:** Hussain is setting up a plumbing business and needs to borrow \$150000 from a bank to buy a truck and other equipment. The bank will charge him interest of 11% p.a., compounded annually. Hussain will pay the whole loan off all at once four years later. 4

   (a) How much will Hussain owe the bank at the end of four years?

Solution: Each year Hussain owes 111% of what he owed the previous year.
Amount at the end of four years $= 150000 \times (1.11)^4$

$$\approx \$227710.56$$

(b) What is the percentage increase in the money owed at the end of four years?

Solution: Final amount = original amount $\times (1.11)^4$

$$\approx \text{ original amount } \times 1.5181$$

So the total increase over four years is about 51.81%.

(c) What is the total interest that Hussain will pay on the loan?

Solution: Total interest $\approx 227710.56 - 150000$

$$= \$77710.56$$

(d) What would the simple interest on the loan have been, assuming the same interest rate of 11% p.a.?

Solution: Simple interest $= PRT$

$$\begin{aligned} &= 150000 \times 0.11 \times 4 \\ &= 66000 \end{aligned}$$

Note: Making no repayments on a loan that is accruing compound interest is a risky business practice because, as this example makes clear, the amount owing grows with increasing rapidity as time goes on. This is particularly relevant to credit card debt.

3. Eleni wants to borrow money for three years to start a business, and then pay all the money back, with interest, at the end of that time. The bank will not allow her final debt, including interest, to exceed \$300000. Interest is 9% p.a., compounded annually. What is the maximum amount that Eleni can borrow? 2

Solution: Each year Eleni will owe 109% of what she owed the previous year.

Hence final debt = original debt $\times 1.09 \times 1.09 \times 1.09$

$$\text{final debt} = \text{original debt} \times (1.09)^3$$

Reversing this, original debt = final debt $\div (1.09)^3$

$$= 300000 \div (1.09)^3$$
$$\approx \$231655$$

7.2 Exercises:

1. Christine invested \$100000 for six years at 5% p.a. interest, compounded annually. 5

   (a) By multiplying by 1.05, find the value of the investment after one year.

   Solution: $\$100000 \times 1.05 = \$105000$

   (b) By multiplying by $(1.05)^2$, find the value of the investment after two years.

   Solution: $\$100000 \times (1.05)^2 = \$110250$

   (c) By multiplying by $(1.05)^6$, find the value of the investment after six years.

   Solution: $\$100000 \times (1.05)^6 \approx \$134009.56$

   (d) Find the percentage increase in the investment over the six years.

   Solution: Percentage increase $= \frac{\$134009.56 - \$100000}{\$100000} \times 100 \approx 34.01\%$

   (e) Find the total interest earned over the six years.

   Solution: Total interest $= \$134009.56 - \$100000 = \$34009.56$

2. Find the simple interest on the principal of \$100000 over the six years at the same rate of 5% p.a. 1

Solution: Simple interest $= \$100000 \times 5\% \times 6 = \$30000$

3. Gary has borrowed \$200000 for six years at 8% p.a. interest, compounded annually, in order to start his indoor decorating business. He intends to pay the whole amount back, plus interest, at the end of the six years. [5]

   (a) Find the amount owing after one year.

   Solution: $\$200000 \times 1.08 = \$216000$

   (b) Find the amount owing after two years.

   Solution: $\$200000 \times (1.08)^2 \approx \$233280$

   (c) Find the amount owing after six years.

   Solution: $\$200000 \times (1.08)^6 \approx \$318111.47$

   (d) Find the percentage increase in the loan over the six years.

   Solution: Percentage increase $= \frac{\$318111.47 - \$200000}{\$200000} \times 100 \approx 59.06\%$

   (e) Find the total interest charged over the six years.

   Solution: Total interest $= \$318111.47 - \$200000 = \$118111.47$

4. Find the simple interest on the principal of \$200000 over the six years at the same rate of 8% p.a. [1]

   Solution: Simple interest $= \$200000 \times 8\% \times 6 = \$96000$

5. A couple take out a housing loan of \$320000 over a period of 20 years. They make no repayments over the 20-year period of the loan. Compound interest is payable at $6\frac{1}{2}\%$ p.a., compounded annually. How much would they owe at the end of the 20-year period, and what is the total percentage increase? [1]

   Solution: Owing amount $= \$320000 \times (1 + 6.5\%)^{20} \approx \$1126970.22$
   Percentage increase $= \frac{\$1126970.22 - \$320000}{\$320000} \times 100 \approx 252.18\%$

6. The population of a city increases annually at a compound rate of 3.2% for five years. If the population is 21000 initially, what is it at the end of the five-year period, and what is the total percentage increase? [1]

Solution: Final population $= 21000 \times (1 + 3.2\%)^5 \approx 24423.60$
Percentage increase $= \frac{24423.60 - 21000}{21000} \times 100 \approx 16.3\%$

7. Suzette wants to invest a sum of money now so that it will grow to \$180000 in 10 years' time. How much should she invest now, given that the interest rate is 6% compounded annually? 1

Solution: Present value $= \$180000/(1.06)^{10} \approx \$100927.08$

8. A bank offers 8% p.a. compound interest. How much needs to be invested if the investment is to be worth \$100000 in: 4

(a) 10 years?

Solution: Present value $= \$100000/(1.08)^{10} \approx \$46319.38$

(b) 20 years?

Solution: Present value $= \$100000/(1.08)^{20} \approx \$21454.70$

(c) 25 years?

Solution: Present value $= \$100000/(1.08)^{25} \approx \$14693.57$

(d) 100 years?

Solution: Present value $= \$100000/(1.08)^{100} \approx \$214.55$

8 Depreciation

This final section is about *things losing value*. Here our percentages reduce the original value of the items:

8.1 Examples:

1. The Medicine Home Delivery Company bought a car four years ago for \$40000, and assumed that the value of the car would depreciate at 20% p.a. 4

(a) What value did the car have at the end of two years?

Solution: The value each year is taken to be $100\% - 20\% = 80\%$ of the value in the previous year.
Value at the end of two years $= 40000 \times 0.80 \times 0.80$

$$= 40000 \times (0.80)^2$$
$$= \$25600$$

(b) What value does the car have now, after four years?

Solution: Value at the end of four years = $40000 \times 0.80 \times 0.80 \times 0.80 \times 0.80$

$$= 40000 \times (0.80)^4$$
$$= \$16384$$

(c) What is the percentage decrease in value over the four years?

Solution: $$\text{Final value} = \text{original value } \times (0.80)^4$$

$$= \text{ original value } \times 0.4096$$

Hence the percentage decrease over four years is $100\% - 40.96\% = 59.04\%$

(d) What is the average depreciation on the car over the four years? (Express your answer in dollars per year)

Solution: Depreciation over four years = $40000 - 16384$

$$= \$23616$$

Average depreciation per year = $23616 \div 4$

$$= \$5904 \text{ per year}$$

2. A school buys new computers every four years. At the end of the four years, it offers them for sale to the students on the assumption that they have depreciated at 35% p.a. (per annum). The school is presently advertising some computers at $400 each. 2

(a) What did each computer cost the school originally?

Solution: Each year a computer is worth $100\% - 35\% = 65\%$ of its value the previous year.
Hence final value = original value $\times 0.65 \times 0.65 \times 0.65 \times 0.65$

$$\text{final value} = \text{original value } \times (0.65)^4$$

Reversing this, original value = final value $\div (0.65)^4$

$$= 400 \div (0.65)^4$$
$$\approx \$2241$$

(b) What is the average depreciation on each computer, in dollars per year?

Solution: Depreciation over four years $\approx 2241 - 400$

$$= \$1841$$

Average depreciation per year $\approx 1841 \div 4$

$$\approx \$460$$

8.2 Exercises:

1. The landlord of a large block of home units purchased washing machines for its units four years ago for \$400000, and is assuming a depreciation rate of 30%. 6
   (a) By multiplying by 0.70, find the value after one year.

   Solution: $\$400000 \times 0.70 = \$280000$

   (b) By multiplying by $(0.70)^2$, find the value after two years.

   Solution: $\$400000 \times (0.70)^2 = \$196000$

   (c) By multiplying by $(0.70)^3$, find the value after three years.

   Solution: $\$400000 \times (0.70)^3 = \$137200$

   (d) By multiplying by $(0.70)^4$, find the value after four years.

   Solution: $\$400000 \times (0.70)^4 = \$96040$

   (e) What is the percentage decrease in value over the four years?

   Solution: Percentage decrease $= (1 - \frac{\$96040}{\$400000}) \times 100 \approx 76\%$

   (f) What is the average depreciation on the washing machines, in dollars p.a. over the four years?

   Solution: Average depreciation $= \frac{\$400000 - \$96040}{4} \approx \$76049$ per year

2. A business spent \$560000 installing alarms at its premises and then depreciated them at 20% p.a. Find the value after five years, and the percentage depreciation of their value. 1

Solution: Value after five years = $\$560000 \times (0.80)^5 \approx \$180224$
Percentage depreciation = $(1 - \frac{\$180224}{\$560000}) \times 100 \approx 67.82\%$

3. The population of a sea lion colony decreases at a compound rate of 2% p.a. for 10 years. If the population is 8000 initially, what is it at the end of the 10-year period? [1]

Solution: Final population = $8000 \times (0.98)^{10} \approx 6612$ sea lions

4. The Northern Start Bus Company bought a bus for \$480000, depreciated it at 30% p.a., and sold it again seven years later for \$60000. Was the price that they obtained better or worse than the depreciated value, and by how much? [1]

Solution: Depreciated value = $\$480000 \times (0.70)^7 \approx \$33684$
The sale price was better by $\$60000 - \$33684 = \$26316$

5. Mr Wong's 10-year-old used car is worth \$4000, and has been depreciating at 22.5% p.a. (Calculate amounts of money in whole dollars.) [5]

   (a) Use division by 0.775 to find how much it was worth a year ago.

   Solution: Value a year ago = $\$4000/0.775 \approx \$5161$

   (b) Find how much it was worth two years ago.

   Solution: Value two years ago = $\$5161/0.775 \approx \$6660$

   (c) Find how much it was worth 10 years ago.

   Solution: Value 10 years ago = $\$4000/(0.775)^{10} \approx \$20860$

   (d) What is the total percentage depreciation on the car over the 10-year period?

   Solution: Percentage depreciation = $(1 - \frac{\$4000}{\$20860}) \times 100 \approx 80.8\%$

   (e) What was the average depreciation in dollars per year over the 10-year period?

   Solution: Average depreciation = $\frac{\$20860 - \$4000}{10} \approx \$1686$ per year

9 Homework

9.1 Review of Percentages

1. Express each percentage as a decimal.

   (a) 175%
   (b) 0.6%
   (c) 142.6%
   (d) $\frac{1}{4}$%
   (e) 56%
   (f) 75%
   (g) $37\frac{1}{2}$%
   (h) $16\frac{2}{3}$%
   (i) 6.4%

2. Express each fraction or decimal as a percentage.

   (a) $\frac{3}{8}$
   (b) $\frac{9}{16}$
   (c) $2\frac{1}{4}$
   (d) $\frac{2}{3}$
   (e) $\frac{4}{3}$
   (f) $\frac{3}{400}$
   (g) 0.04
   (h) 0.015
   (i) 1.175

3. Evaluate these amounts, correct to 2 decimal places where necessary.

   (a) 26% of 264
   (b) 138.5% of 650
   (c) 150% of 846

4. Evaluate these amounts, correct to the nearest cent where necessary.

   (a) 23.7% of $960
   (b) 3.2% of $1500
   (c) 0.25% of $800
   (d) $6\frac{1}{2}$% of $200
   (e) $\frac{1}{4}$% of $840
   (f) 7.25% of $1600

5. Find what percentage the first quantity is of the second quantity, correct to 1 decimal place.

   (a) 7 km, 50 km
   (b) $4, $200
   (c) 14 kg, 400 kg

6. Find what percentage the first quantity is of the second quantity, correct to 2 decimal places where necessary. You will need to express both quantities in the same unit first.

   (a) 3.4 cm, 2 m
   (b) 8 hours, 2 weeks
   (c) 250 m, 4 km
   (d) 1 day, 1 year
   (e) 33 weeks, 1 century
   (f) 5 apples, 16 dozen apples

7. A sample of a certain alloy weighs 1.6 g.

   (a) Aluminium makes up 48% of the alloy. What is the weight of the aluminium in the sample?
   (b) The percentage of lead in the alloy is 0.23%. What is the weight of the lead in the sample?

8. Carbon dioxide makes up 0.059% of the mass of the Earth's atmosphere. The total mass of the atmosphere is about 5 million megatonnes. What is the total mass of the carbon dioxide in the atmosphere?

9.2 Using Percentages

1. What percentage of the total cost is a deposit of:

   (a) \$33 on a television valued at \$550 ?

   (b) \$124.10 on a stove valued at \$1460 ?

2. Find the selling price if the commission and the commission rate are as given.

   (a) Commission \$35, rate 7%

   (b) Commission \$646, rate 3.4%

   (c) Commission \$16586.96, rate 5.2%

   (d) Commission \$3518.61, rate 11.4%

3. Find, to 1 decimal place, the percentage profit or loss on costs in these situations.

   (a) Costs \$16000 and sales \$18000

   (b) Costs \$162000 and sales \$150000

   (c) Costs \$2800000 and sales \$3090000

   (d) Costs \$289000000 and sales \$268000000

9.3 Simple Interest

1. Calculate the missing entries for these simple interest investments.

	Principal	Rate p.a.	Time in years	Total interest
a	\$10000	8%		\$3200
b	\$4400000	$7\frac{1}{2}$%		\$3960000
c	\$5000		6	\$1350
d	\$260000		8	\$83200
e		6%	5	\$900
f		3.6%	4	\$115.20

2. Sartoro invested \$80000 in a building society that pays 6.5% p.a. simple interest. Over the years, the investment has paid him \$57200 in interest. How many years has he had the investment?

3. Madeline has received \$168000 in total simple interest payments on an investment of \$400000 that she made six years ago. What rate of interest has the bank been paying?

4. An investor wishes to earn \$240000 interest over a five-year period from an account that earns 12.5% p.a. simple interest. How much does the investor have to deposit into the account?

9.4 Percentage Increase & Decrease

1. Rainfall across one state has decreased over the last five years by about 24%. By multiplying by $76\% = 0.76$, estimate, correct to the nearest 10 mm, the annual rainfall this year at a place where the rainfall five years ago was:

   (a) 1000 mm

   (b) 250 mm

   (c) 680 mm

   (d) 146 mm

2. Admissions to different wards of St Luke's Hospital mostly rose from 2006 to 2007, but by quite different percentage amounts. Find the percentage increase or decrease in wards where the numbers during 2006 and 2007, respectively, were:

(a) 50 and 68
(b) 120 and 171
(c) 92 and 77
(d) 24 and 39

3. Radix Holdings Pty Ltd recently issued bonus shares to its shareholders. Each shareholder received an extra 12% of the number of shares currently held. Find the original holding of a shareholder who now holds:

(a) 672 shares
(b) 4816 shares
(c) 1000 shares
(d) 40200 shares

4. A research institute is trying to find out how much water Lake Grendel had 1000 years ago. The lake now contains 24000 megalitres, but there are various conflicting theories about the percentage change over the last 1000 years. Find how much water the lake had 1000 years ago, correct to the nearest 10 megalitres, if in the last 1000 years the volume has:

(a) risen by 80%
(b) fallen by 28%
(c) risen by 140%
(d) fallen by 4%

9.5 Repeated Increase & Decrease

1. Shares in the Metropolitan Brickworks have been falling by 23% per year for the last five years. Find the present worth of a parcel of shares whose original worth five years ago was:

(a) $1000
(b) $120000
(c) $25660
(d) $3860000

2. (a) A shirt is discounted by 50% and the resulting price is then increased by 50%. By what percentage is the price increased or decreased from its original value?

   (b) The price of a shirt is increased by 50% and the resulting price is then decreased by 50%. By what percentage is the price increased or decreased from its original value?

   (c) Can you explain the relationship between your answers to parts **a** and **b** ?

3. A book shop has a 50% sale on all stock, and has a container of books with the sale price reduced by a further factor of 16%.

   (a) What was the total discount on each book in the container?

   (b) If a book in the container is now selling for $10.50, what was its original price?

4. A particular strain of bacteria increases its population on a certain prepared Petri dish by 34% every hour. Calculate the size of the original population four hours ago if there are now 56000 bacteria.

9.6 Compound Interest

(a) Find the compound interest on \$1000 at 5% p.a., compounded annually for 200 years.

(b) Find the simple interest on \$1000 at 5% p.a. for 200 years.

1. \$10000 is borrowed for five years and compound interest at 10% p.a. is charged by the lender.

   (a) How much money is owed to the lender after the five-year period?

   (b) How much of this amount is interest?

2. Money borrowed at 8% p.a. interest, compounded annually, grew to \$100000 in four years.

   (a) Find the total percentage increase.

   (b) Hence find the original amount invested.

3. A man now owes the bank \$56000, after having taken out a loan five years ago. Find the original amount that he borrowed if the rate of interest per annum, compounded annually, has been:

   (a) 3%

   (b) 5.6%

   (c) 9.25%

   (d) 15%

4. Ms Smith invested \$50000 at 6% p.a. interest, compounded annually, for four years. The tax department wants to know exactly how much interest she earned each year. Calculate these figures for Ms Smith.

5. Mrs Robinson has taken out a loan of \$300000 at 8% p.a. interest, compounded annually, for four years. She wants to know exactly how much interest she will be charged each year so that she can include it as a tax deduction in her income tax return. Calculate these figures for Mrs Robinson.

9.7 Depreciation

1. The Hungry Hour Cafe purchased an air-conditioning system six years ago for \$250000, and is assuming a depreciation rate of 20%.

   (a) Find the value after one year.

   (b) Find the value after two years.

   (c) Find the value after six years.

   (d) What is the percentage decrease in value over the six years?

   (e) What is the average depreciation, in dollars p.a., on the air-conditioning system over the six years?

2. The Backyard Rubbish Experts bought a fleet of small trucks for \$1340000 and depreciated them at 22.5% p.a. Five years later they sold them for \$310000. Was the price that they obtained better or worse than the depreciated value, and by how much?

3. A landlord spent \$3400 on vacuum cleaners for his block of home units and depreciated them for taxation purposes at 25% p.a. Find their value at the end of each of the first three years, and the amount of the depreciation that the landlord could claim against his taxable income for each of those three years.

4. Lara and Kate each received \$100000 from their parents. Lara invested the money at 6.2% p.a. compounded annually, whereas Kate bought a luxury car that depreciated at a rate of 20% p.a. What were the values of their investments at the end of five years?

5. Taxis depreciate at 50% p.a., and other cars depreciate at 22.5% p.a.

    (a) What is the total percentage depreciation on each type of vehicle after seven years?

    (b) What is the difference in value, to the nearest dollar, after seven years of a fleet of taxis and a fleet of other cars, if both fleets cost \$1000000 ?