Year 9 Mathematics
Topic 5
Linear Equations & Inequalities

PEN Education

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1 Introduction

You are now equipped with the skill to manipulate and simplify algebraic expressions from Lessons 1 and 4. As such, today we are going to practise the skill of converting real life problems into algebraic ones and then solving these problems mathematically.

Let us immediately begin with 2 examples to gauge the flavour of today's lesson:

1.1 Examples:

1. Fiona is 5 cm taller than Tristan. Tristan's height is h cm. What is Fiona's height? [1]

2. S euret weighs 2 kg less than his older sister Vivian. If Vivian weighs w kg, what is Seuret's weight? [1]

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2 Constructing Expressions

Before we continue further it is important that our vocabularies are the same:

Definition 1

Expression:

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Definition 2

Linear:

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You will want to construct *linear expressions* to solve the following

2.1 Exercises:

1. W hen Andriana's age is doubled, the number is 3 more than Helen's age. If Andriana's age is x years, what is Helen's age? [1]

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2. T he length of a rectangle is 5 metres more than its width. [2]

   (a) If w metres is the width of the rectangle, express the length of the rectangle in terms of w.

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(b) If ℓ metres is the length of the rectangle, express the width of the rectangle in terms of ℓ.

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3. I n a competition, Deeksha scored 18 points more than Greta and Deirdre scored 5 points less than twice the number of points Greta scored. If a is the number of points Greta scored: [2]

   (a) express the number of points Deeksha scored in terms of a

   (b) express the number of points Deirdre scored in terms of a

4. M atch each of the following mathematical expressions with its corresponding verbal expression. [9]

   (a) $4 + 2x$

   Six less than four times x

   $x - 5$

   Three times one more than x

   $2x - 4$

   Two less than one-quarter of x

   $3(x + 1)$

   One-quarter of two less than x

   $4x - 6$

   Four less than twice x

   $\frac{x}{4} - 2$

   Six more than half of x

   $\frac{x-2}{4}$

   One more than three times x

   $x + 6$

   Five less than x

   $3x + 1$

   Six more than x

   $\frac{x}{2} + 6$

   Four more than twice x

3 Solving simple linear equations

Definition 3

Equation:

. . . . . .
. . . . . .
. . . . . .
. . . . . .

Definition 4

Equality of equations:

. . . . . .
. . . . . .
. . . . . .
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Allow us to consider the following set of equations:

$$2x + 7 = 11$$
$$2x + 9 = 13$$

Equation (2) is obtained from equation (1) by adding 2 to each side of the equation.

So equation (1) is obtained from equation (2) by subtracting 2 from each side of the equation.

Equations (1) and (2) are said to be equivalent equations.

Equation (3), below, is obtained from equation (1) by subtracting 7 from each side of the equation.

$$2x = 4$$
$$x = 2$$

Equation (4) is obtained from equation (3) by dividing each side of the equation by 2. You can obtain equation (3) from equation (4) by multiplying each side of the equation by 2 .

All of the above equations are equivalent.

3.1 Examples:

1. S olve: 4

(a) $3x + 5 = 20$ (b) $5x - 7 = 18$ (c) $3 - 2x = 15$ (d) $-3p = \frac{2}{5}$

2. S olve: 2

(a) $3x + 7 = 2x + 13$ (b) $5a - 21 = 14 - 2a$

1. S olve these equations. 14

(a) $a + 2 = 5$

(b) $d - 15 = 3$

(c) $9q = -27$

(d) $x - 6 = 5\frac{1}{3}$

(e) $2a + 5 = 7$

(f) $5d - 7 = 23$

(g) $5h + 21 = 11$

(h) $3a - 16 = -31$

(i) $2x + 11 = 7\frac{1}{4}$

(j) $2 - 3a = 8$

(k) $3 - 5d = -22$

(l) $5x + 5 = 3x + 1$

(m) $5x - 6 = x - 2$

(n) $4x - 3 = 18 - 3x$

4 Equations with brackets

You will want to pay attention to this section. The reason for this is that you are accustomed to *always expanding brackets*, but now we sometimes do not!

Solve:

a)

$$3(x+2) = 21$$

Solution

$$\begin{aligned} x+2 &= 7 \quad \text{(Divide both sides by 3.)} \\ x &= 5 \end{aligned}$$

b)

$$2(3-x) = 12$$

Solution

$3 - x = 6$ (Divide both sides by 2.)

$-x = 3$

$$x = -3$$

4.1 Examples:

1. S olve $3(x+5) = 31$ [1]

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   .....
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2. S olve: [3]

   (a) $2(x+1) + 4(x+3) = 26$

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   .....
   .....

   (b) $3(a+5) = 2(a+6)$

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   .....
   .....

   (c) $3(y-3) - 2(y-4) = 4$

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4.2 Exercises:

1. S olve for x. 12

(a) $2(x+3) = 8$

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(b) $5(x+1) = 10$

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(c) $-3(2x-6) = 12$

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(d) $2(x+3) = 15$

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.....
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(e) $4(7-x) = 7$

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.....
.....

(f) $5(x-3) = \frac{2}{3}$

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.....

(g) $2(a+1)+4(a+2) = 22$

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(h) $5(c+2)-2(c+1) = 17$

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(i) $2(x+3)-3(x-4) = 20$

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(j) $5(a+3) = 3(2a+1)$

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(k) $-2(x+4)+3(x-2) = 16$

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(l) $\frac{1}{2}(2x+5)+6(x-2) = 4\frac{1}{2}$

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5 Linear Equations involving fractions

When there are fractions in equations, the standard procedure is to remove the fractions by multiplying both sides of the equation by an appropriate whole number. The next step is to remove the brackets.

1. S olve: [2]

   (a) $2x + \frac{1}{2} = \frac{2}{3}$

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   (b) $3x - \frac{1}{4} = \frac{4}{5}$

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2. S olve: [2]

   (a) $\frac{2x}{3} + \frac{1}{5} = 4$

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   (b) $10 - \frac{a+3}{4} = 6$

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3. S olve $2.1x + 3.5 = 9.4$ [1]

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4. S olve $\frac{2x}{3} - 3 = x + \frac{3}{4}$ [1]

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5. S olve $\frac{a+5}{4} = \frac{a+3}{3}$ [1]

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Crash Course:

- Remove all fractions by multiplying both sides of the equation by the lowest common multiple of the denominators.
- Remove all brackets.
- Collect like terms and solve the equation.

5.1 Exercises:

1. S olve for the unknown pronumeral: [9]

(a) $2x - \frac{1}{2} = \frac{1}{4}$

(b) $\frac{7}{2} - 3y = \frac{2}{3}$

(c) $-2x + \frac{1}{3} = \frac{1}{5}$

(d) $\frac{a}{3} + 5 = 3$

(e) $\frac{3b}{7} + 6 = 2$

(f) $\frac{2}{3}(m - 3) = 1$

(g) $\frac{2y+1}{3} + 4 = 7$

(h) $\frac{3y-1}{2} + 2 = 9$

(i) $2 + \frac{2x-1}{5} = 5$

(j) $e + 1.8 = 2.9$

(k) $1.2u = 15.6$

(l) $1.2x + 4 = 10$

(m) $\frac{3x}{4} - 1 = \frac{x}{2} + 3$

(n) $\frac{x}{3} - 4 = 6 - \frac{2x}{5}$

(a) $1.6x + 10 = 0.9x + 12$

(b) $4.8 - 1.3x = 23 + 1.3x$

(c) $\frac{a+3}{2} = \frac{a+1}{5}$

(d) $\frac{2a+1}{3} = \frac{3a+1}{4}$

(e) $\frac{3a+2}{4} + 2 = a$

(f) $\frac{3a-2}{4} = \frac{a-5}{2}$

(g) $\frac{4a-1}{3} + a = 2$

(h) $5a + 9 = 24$

(i) $\frac{c}{3} - 2 = 4$

(j) $\frac{4e}{3} + \frac{1}{2} = 2$

(k) $2g + 5 = 7g - 6$

(l) $2(i - 1) = 5(i + 6)$

(m) $2(k + 1) - 3(k - 2) = 7$

(n) $\frac{m+1}{3} = \frac{m-2}{5}$

(o) $\frac{2q-2}{5} + 1 = 4q$

6 Outro

Just as when you are driving, it is important to know the roadmap what lies ahead. It is equally important to be able to put your studies into to context too.

Next week we are continueing with this same topic, creating more sophisticated equations and introducing the idea of an *inequality*.

Here is what the table of contents will look like next week:

1. Introduction (as always)
2. Using Linear Equations to solve problems
3. Literal Equations
4. $\Rightarrow$ Inequalities $\Leftarrow$
5. Solving LINEAR INEQUALITIES
6. Homework (as usual)

7 Homework

7.1 Constructing Expressions

1. T he length of a rectangular paddock is 20 m less than three times its width. If the width of the paddock is x m, express the length of the paddock in terms of x. [1]

2. I n a triathlon race, Luca ran at an average speed 5 times his average swimming speed. Also, when his average running speed was multiplied by 4 , this number was 3 less than his average speed for the cycling leg. [1]

3. I f x km/h is Luca's average swimming speed, find expressions in terms of x for his: [2]

   (a) average running speed

   (b) average cycling speed

4. G emma is 6 cm shorter than Gavin and 4 cm taller than Brent. If x cm represents Gemma's height, express: [2]

   (a) Gavin's height in terms of x

   (b) Brent's height in terms of x

7.2 Solving simple linear equations

1. S olve these equations. [14]

(a) $b + 7 = 19$

(b) $2a = 6$

(c) $b + 7 = 29\frac{1}{2}$

(d) $-4y = \frac{8}{9}$

(e) $3b + 4 = 19$

(f) $4f - 3 = 13$

(g) $6a + 17 = -1$

(h) $7b - 17 = -66$

(i) $4m - 9 = -13\frac{4}{5}$

(j) $3 - 4b = 15$

(k) $-6 - 7e = 15$

(l) $7x + 15 = 2x + 20$

(m) $4x + 7 = x - 2$

(n) $2x - 3 = 7 - x$

7.3 Equations with brackets

1. (a) $3(x - 2) = 15$ 12

(b) $3(5 - x) = 9$

(c) $-4(3x - 7) = -8$

(h) $4(2d+1)-5(d-2)=17$

(d) $5(x-2)=16$

(i) $5(2\mathrm{y}-3)-3(\mathrm{y}-5)=21$

(e) $-2(x-5)=7$

(j) $5(2a-1)=2(3a+2)$

(f) $2(2x-6)=\frac{2}{6}$

(k) $-5(x+3)-4(x+1)=17$

(g) $4(b-1)+3(b+2)=30$

(l) $\frac{1}{2}(4x+1)+2(x-2)=13$

7.4 Linear Equations involving fractions

1. (a) $3x+\frac{3}{2}=\frac{5}{3}$ [12]

(b) $\frac{2}{3}+4x=\frac{2}{3}$

(e) $2-\frac{x}{3}=6$

(c) $\frac{2}{3}+3x=\frac{1}{5}$

(f) $3\left(\frac{m}{5}+2\right)=2$

(d) $\frac{3a}{4}-\frac{4}{5}=\frac{2}{3}$

(g) $\frac{2x+5}{3}=9$

(k) $3.6r = 9$

(h) $\frac{2x-3}{5} = 3$

(l) $3.8x - 7 = 8.2$

(i) $18 - \frac{7x+2}{3} = 8$

(m) $\frac{x}{3} + 2 = \frac{4x}{3} + 3$

(j) $f + 3.6 = 7.5$

(n) $\frac{5}{3} - \frac{x}{2} = \frac{3x}{4} + \frac{7}{6}$

(a) $5.9x - 7 = 2.4x + 35$

(b) $1.5x + 3.9 = 6.7 - 0.5x$

(f) $\frac{2a-1}{3} - 2 = \frac{a+3}{4}$

(c) $\frac{a+1}{3} = \frac{2a-1}{7}$

(g) $\frac{a}{2} + \frac{a-1}{3} = \frac{a+1}{4}$

(d) $\frac{a+1}{2} + 1 = \frac{a-1}{5}$

(h) $3b - 7 = 32$

(e) $\frac{2a+1}{2} + \frac{a}{3} = 4$

(i) $\frac{d}{2} + 6 = 3$

(l) $3(j+2) = 2(2j-1)$

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(j) $\frac{2f}{3} - 1 = \frac{3}{7}$

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(k) $4h - 2 = 5 - 3h$

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(m) $\frac{2n-1}{3} = \frac{4n+1}{5}$

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(n) $\frac{2r+1}{3} + 2 = \frac{3r-1}{4}$

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Marker's use only.

SECTION	1	2	3	4	5	6	HW	**Total**
MARKS	$\overline{2}$	$\overline{14}$	$\overline{20}$	$\overline{16}$	$\overline{16}$	$\overline{0}$	$\overline{44}$	$\overline{112}$