Year 9 Mathematics
Topic 5b
Linear Equations & Inequalities

PEN Education

2024

Contents

1 Introduction

Welcome to the second installment of Linear Equations and Inequalities. In this lesson we will be applying our techniques to more heavily worded problems; problems that do not immediately seem to be mathematical in nature.

Here are the **buzz-words** that you should be familiar with by the end of the lesson. If you feel uncomfortable when reading any of these words out loud to yourself, ask your tutor for a definition!

linear inequalities equations expressions equality linear equation

2 Using linear equations to solve problems

Let us first practise the skill of converting worded problems to mathematical ones with appropriate variables:

2.1 Examples:

1. T hree children earn weekly pocket money. Andrew earns $2 more than Gina, and Katya earns twice the amount Gina earns. The total of the weekly pocket money is $22. 2

   (a) How much money does Gina earn?

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   (b) How much money do Andrew and Katya earn?

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2. A li and Jasmine each have a number of swap cards. Jasmine has 25 more cards than Ali, and in total the two children have 149 cards. 2

   (a) How many cards does Ali have?

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   (b) How many cards does Jasmine have?

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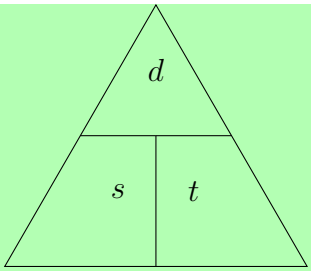
3. Speed is one of the most familiar rates. In problems involving speed, we use the relationship: □

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

   which we can remember conveniently with the following triangle:



4. F or a training run, a triathlete covers 50 km in $4\frac{1}{4}$ hours. She runs part of the way at a speed of 10 km/h, cycles part of the way at a speed of 40 km/h and swims the remaining distance at a speed of $2\frac{1}{2}$ km/h. The athlete runs for twice the time it takes to complete the cycle leg. How long did she take to complete the cycle leg? [2]

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2.2 Exercises:

1. J acques thinks of a number x. When he adds 17 to his number, the result is 32 . What is the value of x ? [1]

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2. W hen 16 is added to twice Simone's age, the answer is 44 . How old is Simone? [1]

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3. W hen 14 is added to half of Suzette's weight in kilograms, the result is 42 . How much does Suzette weigh? [1]

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4. D erek is presently 20 years older than his daughter, Alana. [4]

   (a) If x represents Alana's present age, express each of the following in terms of x.

      i. Derek's present age

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.....

ii. Alana's age in 12 years' time

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iii. Derek's age in 12 years' time

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(b) If Derek's age 12 years from now is twice Alana's age 12 years from now, find their present ages.

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5. A lan, Brendan and Calum each have a number of plastic toys from a fast food store. Brendan has 5 more toys than Alan, and Calum has twice as many toys as Alan. 4

(a) If x represents the number of toys Alan has, express each of the following in terms of x :

i. the number of toys Brendan has

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.....

.....

ii. the number of toys Calum has

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iii. the total number of toys the three boys have

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(b) If the boys have 37 toys in total, determine how many toys each boy has.

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3 Literal Equations

Sometimes instead of having an equation such as $3x + 7 = 22$, we have many variables such as $ax + b = c$. But our algebraic rules still apply and we can treat these letters the same way as we would treat the numbers:

1. Subtract b from both sides
2. Divide both sides by a

Which would have been the same as subtracting both sides by 7 and then dividing by 3 in our *numerical* example.

This type of mathematics - rearranging for a variable even when the other variables are unknown is very helpful in the mathematics that you will soon do. Practise this skill thoroughly:

3.1 Examples:

1. R ewrite $a(x + b) = c$ in terms of x. 1

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   .....
   .....

2. S olve $mx - n = nx + m$ for x. 1

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   .....
   .....

3.2 Exercises:

1. Solve each of these equations for x.

(a) $x + b = c$

(b) $p - x = q$

(c) $cx = b$

(d) $a(x + b) = c$

(e) $\frac{x}{a} = b$

(f) $\frac{x}{a} + b = c$

(g) $\frac{ax}{b} + c = d$

(h) $\frac{mx - n}{n} = m$

(i) $\frac{x}{a} - \frac{a}{b} = b$

(j) $ax + b = cx + d$

(k) $a(x + b) = cx + d$

4 Inequalities

This is the punchline of today's lesson, and an important new concept for you all to grasp: **Inequalities**. The symbols for them are:

1. $> =$ _____
2. $< =$ _____
3. $\leq =$ _____
4. $\geq =$ _____

4.1 Examples:

1. 4_____6 [1]
2. 5_____3 [1]
3. 2_____2 [1]
4. -2_____ -10 [1]
5. 8_____ -8 [1]

Let us now visualise these inequalities on a number line:

1. $x < 2$: [1]

2. $x \geq -1$ [1]

Here is some more practise:

1. G raph each set on the number line. [3]
   (a) $\{x : x \leq -2\}$
   (b) $\{x : x > 3\}$

(c) $\left\{x : x \geq 3\frac{1}{2}\right\}$

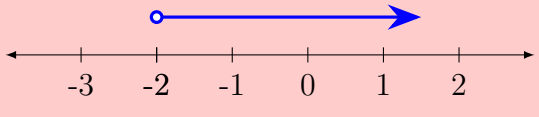
4.2 Exercises:

1. C opy and insert $>$ or $<$ to make each statement true. [3]
   (a) $7\_\_\_\_\_2$
   (b) $-54\_\_\_\_\_-500$
   (c) $21\_\_\_\_\_40$

2. C opy and insert $\leq$ or $\geq$ to make each statement true. [3]
   (a) $-7\_\_\_\_\_-2$
   (b) $-10\_\_\_\_\_-50$
   (c) $12\_\_\_\_\_26$

3. U se set notation to describe each interval. [5]

(a)



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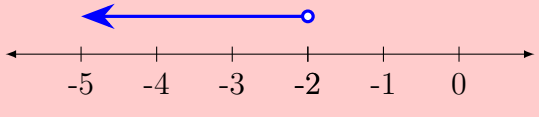

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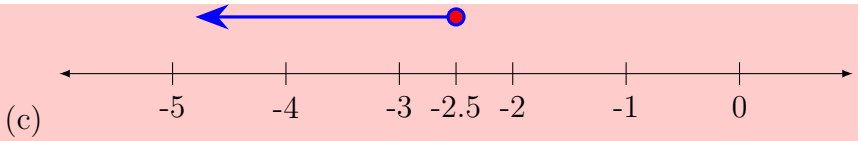


5 Solving linear inequalities

You should now understand the difference between equalities $x = 5$ and inequalities $x < 2$, where the first expression only has one answer _____, and the other has an infinite amount of answers: _____.

We can now add another layer of complexity to this and learn how to do some mathematical operations on on inequality expression.

Let us begin with $-1 < 5$, which is something we agree to be true.

From here we can add 3 to both sides yielding _____. We agree that this is also true!

What about subtraction? Let's continue with our inequality from the last blank and subtract 1: $1 < 7$. That still checks out, very cool.

How about we multiply the whole expression by -1?

$$-1 < -7$$

Is this still true?...

_____!

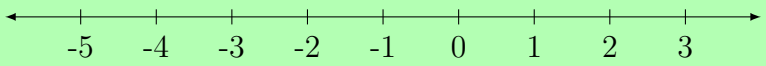
That is the most important thing to remember when manipulating inequalities!

When multiplying an inequality by a negative number you must flip the inequality!

Let us practise our 4 operations of addition, subtraction, multiplication and division on inequality expressions:

1. (a) Solve the inequality $4x - 5 < 3$ 2

   (b) Graph the solution set on the number line



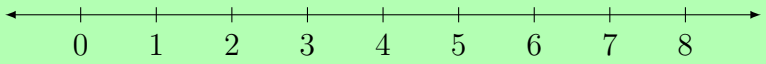
2. S olve each of the following inequalities: 2

(a) $-2x \leq 6$

(b) $-\frac{x}{3} > 4$

3. (a) Solve the inequality $2x + 3 < 3x - 4$. 2

(b) Graph the solution set on the number line.



5.1 Exercises:

1. S olve each of these inequalities. Graph each solution on a number line. 6

(a) $x + 3 \geq 7$

(b) $x - 10 > -6$

(c) $3x > -15$

2. S olve the following purely algebraically: [3]

(a)

$$2x + 1 \geq 5$$

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.....
.....
.....

(b)

$$\frac{4x}{7} \leq -2$$

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.....
.....
.....

(c)

$$\frac{x}{3} - \frac{1}{2} \geq 1$$

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3. S olve: (don't forget to reverse the inequality sign when dividing the expression through by a negative number!) [3]

(a)

$$-4x \leq 20$$

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.....
.....

(b)

$$-\frac{x}{2} \leq 5$$

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(c)

$$-\frac{x}{12} \geq -8$$

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4. S olve: 2

(a)

$$3 - 2x > 5$$

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.....

(b)

$$6 - \frac{2x}{3} < 4$$

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.....
.....

(c)

$$\frac{x+3}{2} \leq \frac{3-x}{2}$$

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5. S olve: 5

(a)

$$1.2x + 6.8 \leq 15.2$$

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.....
.....

(b)

$$1.6(x+7) \leq 1.5(x-3)$$

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(c)

$$2x - 14 \leq 8$$

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(d)

$$\frac{2x+1}{6} > -3$$

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(e)

$$\frac{x-8}{2} - \frac{2x}{3} \geq 3$$

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6. W hen 5 is added to twice p, the result is greater than 17 . What values can p take? [2]

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7. W hen 16 is subtracted from half of q, the result is less than 18 . What values can q take? [2]

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8. W hen $2p$ is subtracted from 10 , the result is greater than or equal to 4 . What values can p take? [2]

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6 Homework

6.1 Using linear equations to solve problems

1. P ing and Anna compete in a handicap sprint race. Anna starts the race 10 m ahead of Ping. Ping runs at an average speed that is 20% faster than Anna's average speed. The two sprinters will be level in the race after 9 seconds. Find the average speed of: [2]

   (a) Anna

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   .....
   .....
   .....

   (b) Ping

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   .....
   .....
   .....

2. Y olan buys 8 pens and receives 80 cents change from $20.00. How much does a pen cost, assuming each pen costs the same amount? [1]

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   .....

3. I f the sum of $2p$ and 19 is the same as the sum of $4p$ and 11 , find the value of p. [1]

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4. I f the sum of half of q and 6 is equal to the sum of one-third of q and 2, find the value of q. [2]

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5. T he length of a swimming pool is 2 m more than four times its width. [3]

   (a) If x metres represents the width of the pool, express the length of the pool in terms of x.

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(b) If the perimeter of the pool is 124 m find the length and width of the pool.

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6. M s Minas earns \$3600 more than Mr Brown, and Ms Lee earns \$2000 less than Mr Brown. [3]

(a) If $\$x$ represents Mr Brown's salary, express the salary of:

i. Ms Minas in terms of x

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ii. Ms Lee in terms of x

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(b) If the total of the three incomes is \$151600, find the income of each person.

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6.2 Literal Equations

1. R ewrite in terms of x: [10]

(a) $-x + m = n$

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.....
.....

(b) $c - bx = e$

(c) $m(nx + p) = n$

(g) $\frac{x}{f} + \frac{g}{h} = k$

(d) $\frac{x+a}{b} = c$

(h) $\frac{x}{b} - b = \frac{a}{b}$

(e) $\frac{mx}{n} = p$

(i) $mx + n = nx - m$

(f) $\frac{ax+b}{c} = d$

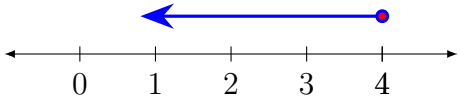
(j) $a(x - b) = c(x - d)$

6.3 Inequalities

1. F ill in the missing blanks with $>$, $<$ or $=$ to make each statement true. [3]
   (a) 3_____-4
   (b) -6_____0
   (c) -2_____5
   (d) 5_____-7
   (e) 0_____0
   (f) 9_____9

2. U se set notation to describe each interval: [3]

(a)



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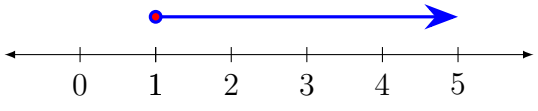

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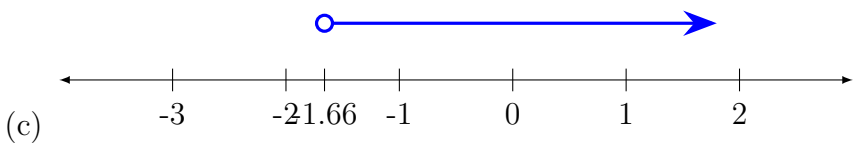

6.4 Solving linear inequalities

1. S olve and also sketch: 3

(a)

$$x - 2 < 3$$

(b)

$$x - 5 > -12$$

(c)

$$\frac{x}{5} \geq 4$$

2. J ust solve: 14

(a)

$$4x - 6 \leq -2$$

(b)

$$3(x + 5) \geq 9$$

(c)

$$\frac{2x}{5} + \frac{1}{4} > 4$$

(d)

$$-10x \geq 130$$

(e)

$$-\frac{x}{7} \geq 4$$

(f)

$$-\frac{x}{2} \geq -8$$

(g)

$$2 - 5x \leq -8$$

(h)

$$4 - \frac{2x}{5} \geq 6$$

(i)

$$\frac{2x-1}{3} - \frac{3x+2}{4} > 3$$

(j)

$$2.4 - 0.7x \leq 12.9$$

(k)

$$2.8(x-4) > 1.3(x+3.5)$$

(l)

$$-5x + 3 \geq 78$$

(m)

$$-\frac{x+2}{3} \leq 7$$

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.....
.....

(n)

$$\frac{x}{4} > -\frac{x+12}{5}$$

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3. T he sum of $4d$ and 6 is greater than the sum of $2d$ and 18 . What values can d take? [2]

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4. A number a is increased by 3 and this amount is then doubled. If the result of this is greater than a, what values can a take? [2]

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Marker's use only.

SECTION	1	2	3	4	5	HW	**Total**
MARKS	$\overline{0}$	$\overline{17}$	$\overline{2}$	$\overline{21}$	$\overline{31}$	$\overline{49}$	$\overline{120}$