Year 9 Mathematics
Topic 5b
Linear Equations & Inequalities

PEN Education

2024

Contents

1 Introduction

Welcome to the second installment of Linear Equations and Inequalities. In this lesson we will be applying our techniques to more heavily worded problems; problems that do not immediately seem to be mathematical in nature.

Here are the **buzz-words** that you should be familiar with by the end of the lesson. If you feel uncomfortable when reading any of these words out loud to yourself, ask your tutor for a definition!

linear inequalities equations expressions equality linear equation

2 Using linear equations to solve problems

Let us first practise the skill of converting worded problems to mathematical ones with appropriate variables:

2.1 Examples:

1. T hree children earn weekly pocket money. Andrew earns \$2 more than Gina, and Katya earns twice the amount Gina earns. The total of the weekly pocket money is \$22. 2

   (a) How much money does Gina earn?

   Solution: Let $\$m$ be the amount of pocket money Gina earns in a week.
   Then Andrew earns $\$(m + 2)$ and Katya earns $\$(2m)$,
   so $m + (m + 2) + 2m = 22$ (The total weekly pocket money is \$22.)

   $$4m + 2 = 22$$

   $4m = 20$

   $$m = 5$$

   So Gina earns \$5 per week.

   (b) How much money do Andrew and Katya earn?

   Solution: Andrew earns \$7 and Katya earns \$10 per week.

2. A li and Jasmine each have a number of swap cards. Jasmine has 25 more cards than Ali, and in total the two children have 149 cards. 2

   (a) How many cards does Ali have?

   Solution: Let x be the number of cards Ali has.
   Jasmine has $(x + 25)$ cards.
   Total number of cards is 149 ,

   $$\text{so } x + (x + 25) = 149$$
   $$2x + 25 = 149$$
   $$2x = 124$$
   $$x = 62$$

   So Ali has 62 cards.

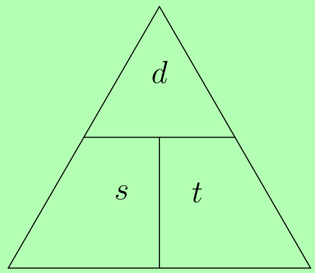
   (b) How many cards does Jasmine have?

Solution: Jasmine has $62 + 25 = 87$ cards.

3. Speed is one of the most familiar rates. In problems involving speed, we use the relationship: ☐

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

which we can remember conveniently with the following triangle:



4. F or a training run, a triathlete covers 50 km in $4\frac{1}{4}$ hours. She runs part of the way at a speed of 10 km/h, cycles part of the way at a speed of 40 km/h and swims the remaining distance at a speed of $2\frac{1}{2}$ km/h. The athlete runs for twice the time it takes to complete the cycle leg. How long did she take to complete the cycle leg? 2

Solution: Let t hours be the time for the cycle leg.

Then $2t$ hours is the time for the running leg and $\left(4\frac{1}{4} - t - 2t\right)$ hours is the time for the swim leg.

Now, distance of run + distance of cycle + distance of swim = total distance,

so $10 \times 2t + 40 \times t + 2\frac{1}{2} \times \left(4\frac{1}{4} - t - 2t\right) = 50$

$$\begin{aligned}
20t + 40t + \frac{5}{2}\left(\frac{17}{4} - 3t\right) &= 50 \\
60t + \frac{85}{8} - \frac{15t}{2} &= 50 \\
480t + 85 - 60t &= 400 \\
420t &= 315 \\
t &= \frac{315}{420} \\
t &= \frac{3}{4}
\end{aligned}$$

The athlete takes $\frac{3}{4}$ hour, or 45 minutes, to complete the cycle leg.

2.2 Exercises:

1. J acques thinks of a number x. When he adds 17 to his number, the result is 32 . What is the value of x ? 1

   Solution:

2. W hen 16 is added to twice Simone's age, the answer is 44 . How old is Simone? 1

   Solution:

3. W hen 14 is added to half of Suzette's weight in kilograms, the result is 42 . How much does Suzette weigh? 1

   Solution:

4. D erek is presently 20 years older than his daughter, Alana. 4

   (a) If x represents Alana's present age, express each of the following in terms of x.

   i. Derek's present age

   Solution:

   ii. Alana's age in 12 years' time

   Solution:

   iii. Derek's age in 12 years' time

   Solution:

   (b) If Derek's age 12 years from now is twice Alana's age 12 years from now, find their present ages.

   Solution:

5. A lan, Brendan and Calum each have a number of plastic toys from a fast food store. Brendan has 5 more toys than Alan, and Calum has twice as many toys as Alan. 4

   (a) If x represents the number of toys Alan has, express each of the following in terms of x :

i. the number of toys Brendan has

Solution:

ii. the number of toys Calum has

Solution:

iii. the total number of toys the three boys have

Solution:

(b) If the boys have 37 toys in total, determine how many toys each boy has.

Solution:

3 Literal Equations

Sometimes instead of having an equation such as $3x + 7 = 22$, we have many variables such as $ax + b = c$. But our algebraic rules still apply and we can treat these letters the same way as we would treat the numbers:

1. Subtract b from both sides
2. Divide both sides by a

Which would have been the same as subtracting both sides by 7 and then dividing by 3 in our *numerical* example.

This type of mathematics - rearranging for a variable even when the other variables are unknown is very helpful in the mathematics that you will soon do. Practise this skill thoroughly:

3.1 Examples:

1. R ewrite $a(x + b) = c$ in terms of x. 1

Solution:

$$x = \frac{c - ab}{a}$$

2. S olve $mx - n = nx + m$ for x. 1

Solution:

$$x = \frac{m + n}{m - n}$$

3.2 Exercises:

1. Solve each of these equations for x.

(a)
$$x + b = c$$

Solution:

(b)
$$p - x = q$$

Solution:

(c)
$$cx = b$$

Solution:

(d)
$$a(x + b) = c$$

Solution:

(e)
$$\frac{x}{a} = b$$

Solution:

(f)
$$\frac{x}{a} + b = c$$

Solution:

(g)
$$\frac{ax}{b} + c = d$$

Solution:

(h)
$$\frac{mx - n}{n} = m$$

Solution:

(i)
$$\frac{x}{a} - \frac{a}{b} = b$$

Solution:

(j)
$$ax + b = cx + d$$

Solution:

(k)
$$a(x + b) = cx + d$$

Solution:

4 Inequalities

This is the punchline of today's lesson, and an important new concept for you all to grasp: **Inequalities**. The symbols for them are:

1. $>$ = **greater than**
2. $<$ = **less than**
3. $\leq$ = **less than or equal to**
4. $\geq$ = **greater than or equal to**

4.1 Examples:

1. 4 ___ $<$ ___ 6 [1]
2. 5 ___ $>$ ___ 3 [1]
3. 2 ___ $=$ ___ 2 [1]
4. -2 ___ $>$ ___ -10 [1]
5. 8 ___ $>$ ___ -8 [1]

Let us now visualise these inequalities on a number line:

1. $x < 2$: [1]

   Solution:

2. $x \geq -1$ [1]

   Solution:

Here is some more practise:

1. G raph each set on the number line. [3]
   (a) $\{x : x \leq -2\}$

   Solution:

   (b) $\{x : x > 3\}$

   Solution:

   (c) $\left\{x : x \geq 3\frac{1}{2}\right\}$

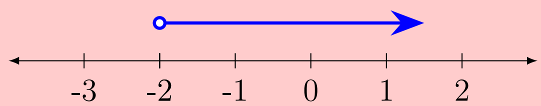
   Solution:

4.2 Exercises:

1. C opy and insert $>$ or $<$ to make each statement true. 3
   (a) $7 \underline{\qquad > \qquad} 2$
   (b) $-54 \underline{\qquad > \qquad} -500$
   (c) $21 \underline{\qquad < \qquad} 40$

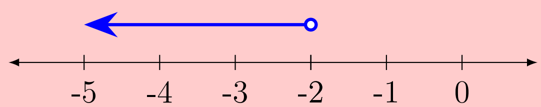
2. C opy and insert $\leq$ or $\geq$ to make each statement true. 3
   (a) $-7 \underline{\qquad \leq \qquad} -2$
   (b) $-10 \underline{\qquad \geq \qquad} -50$
   (c) $12 \underline{\qquad \leq \qquad} 26$

3. U se set notation to describe each interval. 5

(a) 

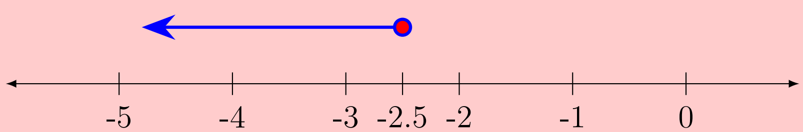
Solution:

$$\{x : x > -2\}$$

(b) 

Solution:

$$\{x : x < -2\}$$

(c) 

Solution:

$$\{x : x \leq -2.5\}$$

5 Solving linear inequalities

You should now understand the difference between equalities $x = 5$ and inequalities $x < 2$, where the first expression only has one answer $\underline{\qquad \mathbf{5} \qquad}$, and the other has an infinite amount of answers: **<u>any number</u> less than 2**.

We can now add another layer of complexity to this and learn how to do some mathematical operations on on inequality expression.

Let us begin with $-1 < 5$, which is something we agree to be true.

From here we can add 3 to both sides yielding $\underline{\quad 2 < 8 \quad}$. We agree that this is also true!

What about subtraction? Let's continue with our inequality from the last blank and subtract 1: $1 < 7$. That still checks out, very cool.

How about we multiply the whole expression by -1?

$$-1 < -7$$

Is this still true?...

NO !

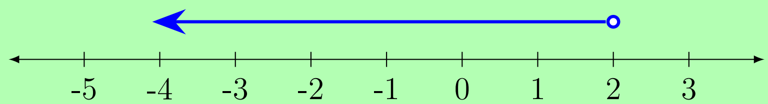
That is the most important thing to remember when manipulating inequalities!

When multiplying an inequality by a negative number you must flip the inequality!

Let us practise our 4 operations of addition, subtraction, multiplication and division on inequality expressions:

1. (a) Solve the inequality $4x - 5 < 3$ 2

   (b) Graph the solution set on the number line



2. S olve each of the following inequalities: 2

   (a)
   $$-2x \leq 6$$

   (b)
   $$-\frac{x}{3} > 4$$

   Solution: $-2x \leq 6$ $x \geq -3$ (Divide both sides by -2 and reverse the inequality sign.)

   Solution: $-\frac{x}{3} > 4$ $x < -12$ (Multiply both sides by -3 and reverse the inequality sign.)

3. (a) Solve the inequality $2x + 3 < 3x - 4$. 2

Solution:

Method 1

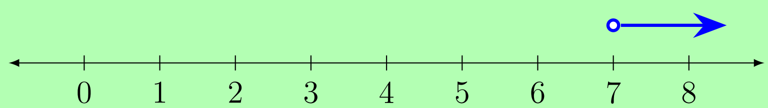
$$\begin{aligned} 2x+3 &< 3x-4 \\ 3 &< x-4 \quad \text{(Subtract } 2x \text{ from both sides.)} \\ 7 &< x \quad \text{(Add 4 to both sides.)} \\ \text{so } x &> 7 \end{aligned}$$

Method 2

$$\begin{aligned} 2x+3 &< 3x-4 \\ 2x &< 3x-7 \\ -x &< -7 \\ x &> 7 \end{aligned}$$

(Subtract 3 from both sides.)
(Subtract $3x$ from both sides.)
(Multiply by -1 and reverse the inequality sign.)
The solution set is $\{x : x > 7\}$.

(b) Graph the solution set on the number line.



5.1 Exercises:

1. S olve each of these inequalities. Graph each solution on a number line. 6

   (a)

   $$x + 3 \geq 7$$

   Solution:

   (b)

   $$x - 10 > -6$$

   Solution:

(c)

$$3x > -15$$

Solution:

2. S olve the following purely algebraically: 3

(a)

$$2x + 1 \geq 5$$

Solution:

(b)

$$\frac{4x}{7} \leq -2$$

Solution:

(c)

$$\frac{x}{3} - \frac{1}{2} \geq 1$$

Solution:

3. S olve: (don't forget to reverse the inequality sign when dividing the expression through by a negative number!) 3

(a)

$$-4x \leq 20$$

Solution:

(b)

$$-\frac{x}{2} \leq 5$$

Solution:

(c)

$$-\frac{x}{12} \geq -8$$

Solution:

4. S olve: 2

(a)

$$3 - 2x > 5$$

Solution:

(b)

$$6 - \frac{2x}{3} < 4$$

Solution:

(c)

$$\frac{x+3}{2} \leq \frac{3-x}{2}$$

Solution:

5. S olve: 5

(a)

$$1.2x + 6.8 \leq 15.2$$

Solution:

(b)

$$1.6(x + 7) \leq 1.5(x - 3)$$

Solution:

(c)

$$2x - 14 \leq 8$$

Solution:

(d)

$$\frac{2x+1}{6} > -3$$

Solution:

(e)

$$\frac{x-8}{2} - \frac{2x}{3} \geq 3$$

Solution:

6. W hen 5 is added to twice p, the result is greater than 17 . What values can p take? [2]

Solution:

7. W hen 16 is subtracted from half of q, the result is less than 18 . What values can q take? [2]

Solution:

8. W hen $2p$ is subtracted from 10 , the result is greater than or equal to 4 . What values can p take? [2]

Solution:

6 Homework

6.1 Using linear equations to solve problems

1. P ing and Anna compete in a handicap sprint race. Anna starts the race 10 m ahead of Ping. Ping runs at an average speed that is 20% faster than Anna's average speed. The two sprinters will be level in the race after 9 seconds. Find the average speed of: 2

   (a) Anna

   Solution:

   (b) Ping

   Solution:

2. Y olan buys 8 pens and receives 80 cents change from \$20.00. How much does a pen cost, assuming each pen costs the same amount? 1

   Solution:

3. I f the sum of $2p$ and 19 is the same as the sum of $4p$ and 11 , find the value of p. 1

   Solution:

4. I f the sum of half of q and 6 is equal to the sum of one-third of q and 2, find the value of q. 2

   Solution:

5. T he length of a swimming pool is 2 m more than four times its width. 3

   (a) If x metres represents the width of the pool, express the length of the pool in terms of x.

   Solution:

   (b) If the perimeter of the pool is 124 m find the length and width of the pool.

   Solution:

6. M s Minas earns \$3600 more than Mr Brown, and Ms Lee earns \$2000 less than Mr Brown. 3

   (a) If $\$x$ represents Mr Brown's salary, express the salary of:

      i. Ms Minas in terms of x

Solution:

ii. Ms Lee in terms of x

Solution:

(b) If the total of the three incomes is \$151600, find the income of each person.

Solution:

6.2 Literal Equations

1. R ewrite in terms of x: [10]

(a) $-x + m = n$

Solution:

(b) $c - bx = e$

Solution:

(c) $m(nx + p) = n$

Solution:

(d) $\frac{x+a}{b} = c$

Solution:

(e) $\frac{mx}{n} = p$

Solution:

(f) $\frac{ax+b}{c} = d$

Solution:

(g) $\frac{x}{f} + \frac{g}{h} = k$

Solution:

(h) $\frac{x}{b} - b = \frac{a}{b}$

Solution:

(i) $mx + n = nx - m$

Solution:

(j) $a(x - b) = c(x - d)$

Solution:

6.3 Inequalities

1. F ill in the missing blanks with >, < or = to make each statement true. [3]

(a) 3____>____ − 4

(b) −6____<____0

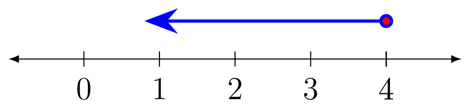
(c) −2____<____5

(d) 5____**5**____ − 7

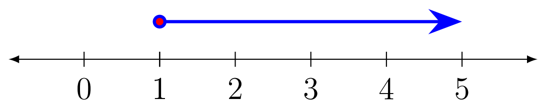
(e) 0____=____0

(f) 9____=____9

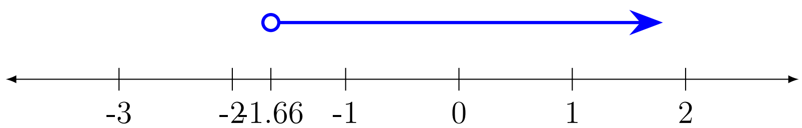
2. U se set notation to describe each interval: [3]

(a)


Solution:

(b)


Solution:

(c)


Solution:

6.4 Solving linear inequalities

1. S olve and also sketch: 3

   (a)

$$x - 2 < 3$$

Solution:

   (b)

$$x - 5 > -12$$

Solution:

   (c)

$$\frac{x}{5} \geq 4$$

Solution:

2. J ust solve: 14

   (a)

$$4x - 6 \leq -2$$

Solution:

(b)

$$3(x+5) \geq 9$$

Solution:

(c)

$$\frac{2x}{5} + \frac{1}{4} > 4$$

Solution:

(d)

$$-10x \geq 130$$

Solution:

(e)

$$-\frac{x}{7} \geq 4$$

Solution:

(f)

$$-\frac{x}{2} \geq -8$$

Solution:

(g)

$$2 - 5x \leq -8$$

Solution:

(h)

$$4 - \frac{2x}{5} \geq 6$$

Solution:

(i)

$$\frac{2x-1}{3} - \frac{3x+2}{4} > 3$$

Solution:

(j)

$$2.4 - 0.7x \leq 12.9$$

Solution:

(k)

$$2.8(x-4) > 1.3(x+3.5)$$

Solution:

(l)

$$-5x + 3 \geq 78$$

Solution:

(m)

$$-\frac{x+2}{3} \leq 7$$

Solution:

(n)

$$\frac{x}{4} > -\frac{x+12}{5}$$

Solution:

3. T he sum of $4d$ and 6 is greater than the sum of $2d$ and 18 . What values can d take? 2

Solution:

4. A number a is increased by 3 and this amount is then doubled. If the result of this is greater than a, what values can a take? 2

Solution:

Marker's use only.

SECTION	1	2	3	4	5	HW	**Total**
MARKS	$\overline{0}$	$\overline{17}$	$\overline{2}$	$\overline{21}$	$\overline{31}$	$\overline{49}$	$\overline{120}$