Year 9 Mathematics
Topic 7a
: Index Laws

PEN Education

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Contents

index	indices	power	root	exponent	reciprocal	sig fig	base	product

1. What is the fourth power of two equal to? [1]

Solution: $2^4 = 16$

2. How do you interpret this? [4]

Solution: $2^4 = 2 \times 2 \times 2 \times 2 = 16$

3. But then what about 2^{-2}? Does this have an answer? What is it's interpretation?! [4]

Solution: $2^4 = 2 \times 2 \times 2 \times 2 = 16$

4. Why are learning about these powers? 1

Solution: Because they express small quantities and large quantites very well. And the world around us expresses itself to us in that way: there are 10^{22} stars in the universe (thereabouts) and the mass of an electron has the mass of 9.1×10^{-31}. These quantities would be dreadful to write out in full. Also, if we study these well we can manipulate them and make our mathematical expressions simpler.

2 The Index Laws

Let us begin simply with the following:

2.1 Examples:

1. Express as a power or as a product of powers. 1
   (a) $5 \times 5 \times 5$

   Solution: $5 \times 5 \times 5 = 5^3$

   (b) $3 \times 3 \times 7 \times 7 \times 7 \times 7$

   Solution: $3 \times 3 \times 7 \times 7 \times 7 \times 7 = 3^2 \times 7^4$

   (c)

2. Express each number as a power of a prime. 1
   (a) 81

   Solution: $81 = 3 \times 3 \times 3 \times 3$

   (b) 128

   Solution: $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 3^4 = 2^7$

Now some definitions:

Definition 1

Base:

Solution: The number 2 in 2^4 is called the base.

Definition 2

Index:

Solution: The number 4 in 2^4 is called the index or exponent.

Laws/Results

Index Laws

Product of Powers

Now with these laws in mind, try the following 10 examples:

2.2 Examples

1. Simplify, expressing the answer in index form. 10

(a) $3^2 \times 3^4$

Solution: $3^2 \times 3^4 = 3^6$

(b) $a^3 \times a^5$

Solution: $a^3 \times a^5 = a^8$

(c) $3x^2 \times x^3$

Solution: $3x^2 \times x^3 = 3x^5$

(d) $2a^2b^3 \times 5ab^2$

Solution: $2a^2b^3 \times 5ab^2 = 10 \times a^{2+1} \times b^{3+2}$

$= 10a^3b^5$

(e) $\frac{3^5}{3^2}$

Solution: $\frac{3^5}{3^2} = 3^{5-2}$

(f) $\frac{9^5}{9^4}$

Solution: $\frac{9^5}{9^4} = 9^1$

(g) $10^6 \div 10^4$

Solution: $10^6 \div 10^4 = 10^{6-4} = 3^3 = 9$
$= 10^2 = 27 = 100$

(h) $a^7 \div a^4$

Solution:
$$a^7 \div a^4 = a^{7-4}$$
$$= a^3$$

(i) $\frac{3y^4}{y}$

Solution: $\frac{3y^4}{y} = 3 \times \frac{y^4}{y}$

(j) $\frac{6x^5}{2x^3}$

Solution: $\frac{6x^5}{2x^3} = \frac{6}{2} \times \frac{x^5}{x^3} = 3 \times y^{4-1}$
$= 3 \times x^{5-3} = 3y^3 = 3x^2$

2. Double marks for each of these: 4

(a)

$$\frac{3x^3y^2}{4xy} \times \frac{6x^2y^3}{x^3y^2}$$

Solution: $\frac{3x^3y^2}{4xy} \times \frac{6x^2y^3}{x^3y^2} = \frac{18x^5y^5}{4x^4y^3} = \frac{9xy^2}{2}$

(b)

$$\frac{8a^2b^3}{3a^3b} \div \frac{4ab^2}{9a^3b^5}$$

Solution: $\frac{8a^2b^3}{3a^3b} \div \frac{4ab^2}{9a^3b^5} = \frac{8a^2b^3}{3a^3b} \times \frac{9a^3b^5}{4ab^2}$

$$= \frac{72 \times a^5 \times b^8}{12 \times a^4 \times b^3}$$

$$= 6ab^5$$

3. And a 1.5× multiplier for these: 6

(a) $\left(\frac{2}{3}\right)^2$

Solution: $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

(b) $\left(\frac{m}{n}\right)^5$

Solution: $\left(\frac{m}{n}\right)^5 = \frac{m^5}{n^5}$

(c) $\left(\frac{x^3}{y^2}\right)^2 \times \left(\frac{y}{x}\right)^4$

Solution: $\left(\frac{x^3}{y^2}\right)^2 \times \left(\frac{y}{x}\right)^4 = \frac{x^6}{y^4} \times \frac{y^4}{x^4}$

(d) $\left(\frac{2x^2}{3}\right)^2 \div \frac{4x^3}{9}$

Solution: $\left(\frac{2x^2}{3}\right)^2 \div \frac{4x^3}{9} = \frac{4x^4}{9} \times \frac{9}{4x^3} = x^2 = x$

(a) $(5a^3)^0$

Solution: $(5a^3)^0 = 1$

(b) $\frac{6x^2y}{xy^2} \times \frac{y^3x}{2y^2x^2}$

Solution:

$$\frac{6x^2y}{xy^2} \times \frac{y^3x}{2y^2x^2} = \frac{6}{2} \times \frac{x^3}{x^3} \times \frac{y^4}{y^4}$$
$$= 3x^0y^0$$
$$= 3 \times 1 \times 1$$
$$= 3$$

(c) $(mn^2)^0$

Solution: $(mn^2)^0 = 1$

(d) $(a^4b^2)^3$

Solution: $(a^4b^2)^3 = a^{12}b^6$

(e) $(2a^4)^3$

Solution: $(2a^4)^3 = 2^3 \times a^{12}$

(f) $2\,(x^2y)^0 \times (x^2y^3)^3$

Solution: $2\,(x^2y)^0 \times (x^2y^3)^3 = 2 \times 1 \times x^6y^9 = 8a^{12} = 2x^6y^9$

(g)

Note: There are different possible interpretations of the word 'simplify'. There may be more than one acceptable simplified form.

2.3 Exercises:

1. State the base and index of: 3

(a) $6^4 =$ ____**6**____ (b) $7^3 =$ ____**7**____ (c) $8^2 =$ ____**8**____

Solution: Base: 6, Index: 4

Solution: Base: 7, Index: 3

Solution: Base: 8, Index: 2

2. Express as a power of a prime number. 3

(a) $8 =$ ____2^3____ (b) $27 =$ ____3^3____ (c) $64 =$ ____2^6____

3. Evaluate: 3

(a) $3^4 =$ **81** (b) $2^7 =$ **128** (c) $5^5 =$ **3125**

4. Express as a product of powers of prime numbers. 3

(a) $18 = 2 \times 3^2$ (b) $24 = 2^3 \times 3$ (c) $144 = 2^4 \times 3^2$

5. Simplify: 3

(a) $2^7 \times 2^3$

Solution: 2^{10}

(b) $3^3 \times 3^4 \times 3^5$

Solution: 3^{12}

(c) $3x^2 \times 4x^3$

Solution: $12x^5$

6. Simplify: 4

(a) $a^2b^3 \times b^2$

Solution: a^2b^5

(c) $2xy^2 \times 3x^2y$

Solution: $6x^3y^3$

(b) $a^3b \times a^2b^3$

Solution: a^5b^4

(d) $4a^3b^2 \times a^2b^4$

Solution: $4a^5b^6$

7. 6

(a) $\frac{3^7}{3^2}$

Solution: 3^5

(d) $\frac{10^{12}}{10^4}$

Solution: 10^8

(b) $\frac{2^6}{2^2}$

Solution: 2^4

(e) $\frac{2x^3}{x^2}$

Solution: $2x$

(c) $10^7 \div 10^2$

Solution: 10^5

(f) $\frac{6x^5}{2x^2}$

Solution: $3x^3$

8. 4

(a) $\frac{a^3b^2}{a^2}$

Solution: ab^2

(c) $\frac{12a^6b^2}{4a^2b}$

Solution: $3a^4b$

(b) $\frac{x^3y^2}{xy}$

Solution: x^2y

(d) $\frac{15xy^3}{3y^2}$

Solution: $5xy$

9. [4]

(a) $\frac{a^3b^2}{ab} \times \frac{a^2b}{a}$

Solution: a^4b^2

(c) $\frac{6ab^2}{5a^3b} \div \frac{12ab}{15a^5b}$

Solution: $3a^2b$

(b) $\frac{x^3y}{xy^2} \times \frac{x^4y^5}{x^2}$

Solution: x^4y^4

(d) $\frac{7x^3y^4}{2xy^2} \div \frac{21x^2y^3}{4x^3y^2}$

Solution: $2x^2y$

10. [6]

(a) $a^4 \times$ ____a^6____ $= a^{10}$

Solution: a^6

(b) $9d^5 \times$ ____**3**____ $= 27d^6$

Solution: $3d$

(c) $15d^7 \div$ ____$5d^5$____ $= 3d^2$

Solution: $5d^5$

(d) $\frac{8ab^4}{24a^2b^6} \times$ ____$3ab^2$____ $=$

Solution: $3ab^2$

(e) $\ell^6m^7 \div$ ____ℓ^4m^2____ $=$ ℓ^2m^5

Solution: ℓ^4m^2

(f) $b^7 \times$ ____b^9____ $= b^{16}$

Solution: b^9

11. [2]

(a) a^0

Solution: 1

Solution: 2

(b) $2x^0$

12. [6]

(a) $3a^0$

Solution: 3

(c) $4a^0 + 3b^0$

Solution: 7

(e) $(4b)^0 + 2b^0$

Solution: 3

(b) $6a^0$

Solution: 6

(d) $6a^0 + 7m^0$

Solution: 13

(f) $(3b)^0 - 5d^0$

Solution: -4

13. Simplify, leaving the answer as a power. [2]

(a) $(2^3)^4$

Solution: 2^{12}

Solution: 3^6

(b) $(3^2)^3$

14. [4]

(a) $(a^3)^2 \times (a^3)^4$

Solution: a^{14}

(c) $2ab^2 \times 3a\,(b^3)^2$

Solution: $6a^2b^8$

(b) $(x^4)^2 \times (x^3)^3$

Solution: x^{17}

(d) $\frac{3ab}{(b^2)^3} \times \frac{4b^7}{3a}$

Solution: $4b^2$

15. [4]

(a) $(3a)^2$

Solution: $9a^2$

(c) $\left(\frac{a}{5}\right)^2$

Solution: $\frac{a^2}{25}$

(b) $(2x)^3$

Solution: $8x^3$

(d) $\left(\frac{2}{x}\right)^3$

Solution: $\frac{8}{x^3}$

16. [4]

(a) $(2a^2b)^2 \times 3ab^3$

Solution: $12a^5b^5$

(c) $(5xy^2)^3 \times (x^2y^3)^2$

Solution: $125x^7y^{13}$

(b) $(3xy^2)^3 \times (x^2y)^2$

Solution: $27x^5y^8$

(d) $(2a^3b)^3 \times 3a^0$

Solution: $24a^9b^3$

17. 4

(a) $\left(\frac{x^2}{y}\right)^2 \times \left(\frac{y^2}{x}\right)^3$

Solution: $\frac{xy^4}{y^2}$

(c) $\frac{(3xy^2)^2 \times (2x^2y)^3}{(6x^2y)^2}$

Solution: x^2y^4

(b) $\left(\frac{4a^2}{b}\right)^2 \times \left(\frac{b}{2a}\right)^3$

Solution: $\frac{8ab}{b^2}$

(d) $\frac{3a^2b^4 \times (2ab^2)^3}{(4a^2b^3)^2}$

Solution: $\frac{3a^2b^2}{4}$

3 Negative Indices

This section is tricky as negative numbers always tend to be. The most important thing you will learn here is:

$$a^{-n} = \frac{1}{a^n}$$

I want you to take a second to realise that this **breaks** your interpretation of a being raised to some power n as being n successive multiplications of the number a.

$$a^n = \underbrace{a \times a \times a \ldots}_{n}$$

Suddenly what does it mean to multiply a by itself $-n$ times?!
Here is some working out space for you to reconcile with this fact.

Solution: Proof:
Consider the product of a^n and a^{-n}:

$$a^n \cdot a^{-n} = a^{n+(-n)} = a^0$$

Since anything raised to the power of 0 is 1, we have:

$$a^0 = 1$$

Therefore:

$$a^n \cdot a^{-n} = 1$$

Dividing both sides by a^n, we get:

$$\frac{a^n \cdot a^{-n}}{a^n} = \frac{1}{a^n}$$

Simplifying the left side, we have:

$$a^{-n} = \frac{1}{a^n}$$

Hence proved.

3.1 Examples:

Worked Example:

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

1. Evaluate [4]

   (a) 6^{-2}

   Solution: $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

   (c) 2^{-7}

   Solution: $2^{-7} = \frac{1}{2^7} = \frac{1}{128}$

   (b) 4^{-3}

   Solution: $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

   (d) 10^{-3}

   Solution: $10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$

2. These ones just reciprocate! [3]

   (a) $\left(\frac{1}{3}\right)^{-1}$

   Solution: $\left(\frac{1}{3}\right)^{-1} = \frac{3}{1} = 3$

   (b) $\left(\frac{2}{7}\right)^{-2}$

   Solution: $\left(\frac{2}{7}\right)^{-2} = \left(\frac{7}{2}\right)^2 = \frac{49}{4}$

   (c) $\left(4\frac{1}{4}\right)^{-2}$

   Solution: $\left(4\frac{1}{4}\right)^{-2} = \left(\frac{17}{4}\right)^{-2} = \left(\frac{4}{17}\right)^2 = \frac{16}{289}$

3. Write as a single power and then evaluate: [8]

   (a) $3^4 \times 3^{-2}$

   Solution: $3^4 \times 3^{-2} = 3^2$

(b) $5^7 \times 5^{-8}$

Solution: $5^7 \times 5^{-8} = 5^{-1}$

$= 9$

(c) $13^{-8} \times 13^{15} \times 13^{-7}$

Solution: $13^{-8} \times 13^{15} \times 13^{-7} = 13^0$

$= 1$

$= 1$

$= \frac{1}{5}$

(d) $\left(\frac{2}{3}\right)^{-6} \times \left(\frac{2}{3}\right)^4$

Solution: $\left(\frac{2}{3}\right)^{-6} \times \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{-2}$

$= \left(\frac{3}{2}\right)^2$

$= \frac{9}{4}$

(e) $\frac{2^4}{2^5}$

Solution: $\frac{2^4}{2^5} = 2^{-1} = \frac{1}{2}$

(g) $\frac{5}{5^3}$

Solution: $\frac{5}{5^3} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

(f) $\frac{3^4}{3^7}$

Solution: $\frac{3^4}{3^7} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

(h) $\frac{3^4}{3^6}$

Solution: $\frac{3^4}{3^6} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

4. Simplify, expressing the answers with positive indices

4

(a) $a^2b^{-3} \times a^{-4}b^5$

Solution: $a^2b^{-3} \times a^{-4}b^5 = a^{2-4} \times b^{-3+5} = \frac{1}{a^2} \times b^2 = \frac{b^2}{a^2}$

(c) $(2a^{-2}b^3)^{-2}$

Solution: $(2a^{-2}b^3)^{-2} = 2^{-2} \times a^4 \times b^{-6} = \frac{1}{2^2} \times a^4 \times \frac{1}{b^6} = \frac{a^4}{4b^6}$

(b) $\frac{x^2y^3}{x^3y^2}$

Solution: $\frac{x^2y^3}{x^3y^2} = x^{-1}y^1 = \frac{1}{x} \times y = \frac{y}{x}$

(d) $\left(\frac{3m^2}{n}\right)^{-4}$

Solution: $\left(\frac{3m^2}{n}\right)^{-4} = \left(\frac{n}{3m^2}\right)^4 = \frac{n^4}{81m^8}$

Note that in general $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$.

3.2 Exercises:

1. Express with a positive index and then evaluate. [4]

(a) 2^{-1}

Solution: $2^{-1} = \frac{1}{2}$
$2^{-1} = 0.5$

(c) 2^{-4}

Solution: $2^{-4} = \frac{1}{2^4}$
$2^{-4} = 0.0625$

(b) 5^{-1}

Solution: $5^{-1} = \frac{1}{5}$
$5^{-1} = 0.2$

(d) 3^{-3}

Solution: $3^{-3} = \frac{1}{3^3}$
$3^{-3} = \frac{1}{27}$

2. Write each fraction as a power of a prime with a negative index. [4]

(a) $\frac{1}{8}$

Solution: $\frac{1}{8} = 2^{-3}$

(c) $\frac{1}{16}$

Solution: $\frac{1}{16} = 2^{-4}$

(b) $\frac{1}{9}$

Solution: $\frac{1}{9} = 3^{-2}$

(d) $\frac{1}{64}$

Solution: $\frac{1}{64} = 2^{-6}$

3. Express with positive indices, evaluating where possible. [4]

(a) a^{-3}

Solution: $a^{-3} = \frac{1}{a^3}$

(b) x^{-7}

Solution: $x^{-7} = \frac{1}{x^7}$

(c) $\frac{3}{a^{-4}}$

Solution: $\frac{3}{a^{-4}} = 3a^4$

(d) $\frac{5}{x^{-5}}$

Solution: $\frac{5}{x^{-5}} = 5x^5$

4. Simplify and then evaluate [4]

(a) $\left(\frac{1}{4}\right)^{-1}$

Solution: $\left(\frac{1}{4}\right)^{-1} = 4$

(b) $\left(\frac{2}{5}\right)^{-2}$

Solution: $\left(\frac{2}{5}\right)^{-2} = \left(\frac{5}{2}\right)^{2}$
$\left(\frac{2}{5}\right)^{-2} = \frac{25}{4}$

(c) $3^5 \times 3^{-2}$

Solution: $3^5 \times 3^{-2} = 3^{5-2}$
$3^5 \times 3^{-2} = 3^3$
$3^5 \times 3^{-2} = 27$

(d) $5^{11} \times 5^{-8}$

Solution: $5^{11} \times 5^{-8} = 5^{11-8}$
$5^{11} \times 5^{-8} = 5^3$
$5^{11} \times 5^{-8} = 125$

5. [4]

(a) $\frac{2^3}{2^6}$

Solution: $\frac{2^3}{2^6} = 2^{3-6}$
$\frac{2^3}{2^6} = 2^{-3}$
$\frac{2^3}{2^6} = \frac{1}{8}$

(b) $\frac{4^2}{4^4}$

Solution: $\frac{4^2}{4^4} = 4^{2-4}$
$\frac{4^2}{4^4} = 4^{-2}$
$\frac{4^2}{4^4} = \frac{1}{16}$

(c) $\frac{8^6}{8^7}$

Solution: $\frac{8^6}{8^7} = 8^{6-7}$
$\frac{8^6}{8^7} = 8^{-1}$
$\frac{8^6}{8^7} = \frac{1}{8}$

(d) $\frac{20^4}{20^6}$

Solution: $\frac{20^4}{20^6} = 20^{4-6}$
$\frac{20^4}{20^6} = 20^{-2}$
$\frac{20^4}{20^6} = \frac{1}{400}$

6. Express with negative index. [3]

(a) $\frac{3}{x}$ (b) $\frac{5}{x^2}$ (c) $\frac{8}{x^4}$

Solution: $\frac{3}{x} = 3x^{-1}$

Solution: $\frac{5}{x^2} = 5x^{-2}$

Solution: $\frac{8}{x^4} = 8x^{-4}$

7. Evaluate. [3]

(a) $\left(\frac{1}{2}\right)^{-1}$

Solution: $\left(\frac{1}{2}\right)^{-1} = 2$

(b) $\left(\frac{2}{3}\right)^{-1}$

Solution: $\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$

(c) $\left(\frac{1}{2}\right)^{-2}$

Solution: $\left(\frac{1}{2}\right)^{-2} = 2^2$
$\left(\frac{1}{2}\right)^{-2} = 4$

8. Simplify, expressing the answer with positive indices. [4]

(a) $x^{-6}y^4 \times x^2y^{-2}$

Solution: $x^{-6}y^4 \times x^2y^{-2} = x^{-6+2}y^{4-2}$
$x^{-6}y^4 \times x^2y^{-2} = x^{-4}y^2$
$x^{-6}y^4 \times x^2y^{-2} = \frac{y^2}{x^4}$

(b) $2a^{-1}b^5 \times 7ab^{-3}$

Solution: $2a^{-1}b^5 \times 7ab^{-3} = 14a^{-1+1}b^{5-3}$
$2a^{-1}b^5 \times 7ab^{-3} = 14b^2$

(c) $\frac{8a^{-4}}{2a^6}$

Solution: $\frac{8a^{-4}}{2a^6} = 4a^{-4-6}$
$\frac{8a^{-4}}{2a^6} = 4a^{-10}$
$\frac{8a^{-4}}{2a^6} = \frac{4}{a^{10}}$

(d) $\frac{36h^{-9}}{9h^{-4}}$

Solution: $\frac{36h^{-9}}{9h^{-4}} = 4h^{-9+4}$
$\frac{36h^{-9}}{9h^{-4}} = 4h^{-5}$
$\frac{36h^{-9}}{9h^{-4}} = \frac{4}{h^5}$

9. Fill in the missing term [6]

(a) $6^4 \times \ldots = 6^2$

Solution: $6^4 \times 6^{-2} = 6^2$

(b) $m^5 \times \ldots = m^{-6}$

Solution: $m^5 \times m^{-11} = m^{-6}$

(c) $d^{-7} \div \ldots = d^{15}$

Solution: $d^{-7} \div d^{-22} = d^{15}$

(d) $(a^5) \cdots = a^{-15}$

Solution: $(a^5)^{-3} = a^{-15}$

(e) $(\ldots)^{-2} = \frac{m^6}{25}$

Solution: $(\frac{m^3}{5})^{-2} = \frac{m^6}{25}$

(f) $(\ldots)^{-2} = p^4q^{-6}$

Solution: $(p^2q^{-3})^{-2} = p^4q^{-6}$

10. Simplify, expressing the answers with positive indices. Evaluate powers where possible. [4]

(a) $(3a^2b^{-2})^3 \times (2a^4)^{-2}$

Solution: $(3a^2b^{-2})^3 \times (2a^4)^{-2} = 27a^6b^{-6} \times \frac{1}{4a^8}$

$(3a^2b^{-2})^3 \times (2a^4)^{-2} = \frac{27}{4}a^{-2}b^{-6}$

$(3a^2b^{-2})^3 \times (2a^4)^{-2} = \frac{27}{4a^2b^6}$

(b) $(6a^5b^{-4})^{-3} \times 2\,(a^3b^{-3})^2$

Solution: $(6a^5b^{-4})^{-3} \times 2\,(a^3b^{-3})^2 = \frac{1}{216a^{15}b^{-12}} \times 2a^6b^{-6}$

$(6a^5b^{-4})^{-3} \times 2\,(a^3b^{-3})^2 = \frac{2}{216}a^{-9}b^6$

$(6a^5b^{-4})^{-3} \times 2\,(a^3b^{-3})^2 = \frac{1}{108a^9b^6}$

(c) $\frac{(2a^4b^{-2})^3}{c^2} \times \frac{(2^2a^{-3}b^2)^{-1}}{c}$

Solution: $\frac{(2a^4b^{-2})^3}{c^2} \times \frac{(2^2a^{-3}b^2)^{-1}}{c} = \frac{8a^{12}b^{-6}}{c^2} \times \frac{c}{4a^3b^{-2}}$

$\frac{(2a^4b^{-2})^3}{c^2} \times \frac{(2^2a^{-3}b^2)^{-1}}{c} = \frac{2a^9b^{-4}}{c^3}$

(d) $\mathbf{j}\frac{(2a^4)^2}{b^7} \div \frac{(a^2)^{-3}}{2b}$

Solution: $\mathbf{j}\frac{(2a^4)^2}{b^7} \div \frac{(a^2)^{-3}}{2b} = \mathbf{j}\frac{4a^8}{b^7} \times \frac{2b}{a^{-6}}$

$\mathbf{j}\frac{(2a^4)^2}{b^7} \div \frac{(a^2)^{-3}}{2b} = \mathbf{j}\frac{8a^{14}}{b^6}$

4 Homework

The Index Laws

1. State the base and index of: [3]

   (a) 10^4

   Solution: Base: 10, Index: 4

   (b) 5

   Solution: Base: 5, Index: 1

   (c) 6^0

   Solution: Base: 6, Index: 0

2. Express as a power of a prime number. [3]

   (a) 243

   Solution: $243 = 3^5$

   (b) 125

   Solution: $125 = 5^3$

   (c) 81

   Solution: $81 = 3^4$

3. Evaluate: [3]

   (a) 7^4

   Solution: 2401

   (b) $2^3 \times 3^5$

   Solution: $8 \times 243 = 1944$

   (c) $6^4 \times 3^2$

   Solution: $1296 \times 9 = 11664$

4. Express as a product of powers of prime numbers. [3]

   (a) 90

   Solution: $90 = 2 \times 3^2 \times 5$

   (b) 700

   Solution: $700 = 2^2 \times 5^2 \times 7$

   (c) 84

   Solution: $84 = 2^2 \times 3 \times 7$

5. Simplify, leaving the answer as a power or a product of powers. [6]

   (a) $3^4 \times 3^5$

   Solution: 3^9

   (b) $3^4 \times 3^7$

   Solution: 3^{11}

   (c) $a^3 \times a^8$

   Solution: a^{11}

   (d) $b^7 \times b^{12}$

   Solution: b^{19}

   (e) $2y \times 3y^4$

   Solution: $6y^5$

   (f) $4b^2 \times 3b^4$

   Solution: $12b^6$

6. Simplify: [2]

(a) $x^2y \times x^3y$

Solution: x^5y^2

(b) $5a^4b \times 2ab^3$

Solution: $10a^5b^4$

7. Simplify, leaving the answer as a power or a product of powers. [6]

(a) $5^4 \div 5$

Solution: 5^3

(b) $7^5 \div 7^3$

Solution: 7^2

(c) $\frac{a^4}{a}$

Solution: a^3

(d) $\frac{a^5}{a^3}$

Solution: a^2

(e) $\frac{10y^{12}}{5y^3}$

Solution: $2y^9$

(f) $\frac{27p^4}{9p}$

Solution: $3p^3$

8. Simplify: [4]

(a)
$$\frac{a^5b^3}{a^4b}$$

Solution: ab^2

(b)
$$\frac{x^4y^7}{x^3y^2}$$

Solution: xy^5

(c)
$$\frac{16a^4b^3}{12a^2b^2}$$

Solution: $\frac{4}{3}a^2b$

(d)
$$\frac{27x^2y^3}{18xy^2}$$

Solution: $\frac{3}{2}xy$

9. Simplify: [4]

(a)
$$\frac{2ab^2}{3a^2b^4} \times \frac{6a^4b^5}{ab}$$

Solution: $4ab^{-1}$

(b)
$$\frac{12x^4y^3}{3x^2y} \times \frac{x^2y^4}{x^3y^5}$$

Solution: $4xy$

(c) $\dfrac{14a^4b^3}{3ab^2} \div \dfrac{7a^5b^4}{6a^3b^5}$

Solution: $8a^2b^{-4}$

(d) $\dfrac{12x^2y}{x^3y^4} \div \dfrac{6xy^2}{x^6y^7}$

Solution: $2x^{-7}y^{-8}$

10. Copy and complete. [9]

(a) $4a^3 \times \_\_\_\_\_ = 12a^7$

Solution: $3a^4$

(b) $a^8 \div \_\_\_\_\_ = a^4$

Solution: a^4

(c) $x^{10} \div \_\_\_\_\_ = x^6$

Solution: x^4

(d) $9d^6 \div \_\_\_\_\_ = 3d$

Solution: $3d^5$

(e) $m^4n^5 \times \_\_\_\_\_ = m^{10}n^7$

Solution: m^6n^2

(f) $a^7b^4 \div \_\_\_\_\_ = a^2b$

Solution: a^5b^3

(g) $14x^5y^2 \times \_\_\_\_\_ = 42x^{10}y^5$

Solution: $3x^5y^3$

(h) $9m^7n^4 \div \_\_\_\_\_ = 3m^2$

Solution: $3m^5n^4$

(i) $18p^2q^6 \div \_\_\_\_\_ = 3pq$

Solution: $6pq^5$

11. Simplify: [2]

(a) xy^0

Solution: x

(b) $7x^0y^0$

Solution: 7

12. Simplify: [6]

(a) $(4a)^0$

Solution: 1

(b) $(3b)^0$

Solution: 1

(c) $(2a+1)^0$

Solution: 1

(d) $(4a+3b)^0$

Solution: 1

(e) $(5m^0+7b)^0$

Solution: 1

(f) $(6m-2c^0)^0$

Solution: 1

13. Simplify, leaving the answer as a power. [2]

(a) $(a^2)^5$

Solution: a^{10}

(b) $(y^5)^6$

Solution: y^{30}

14. Simplify 1

$$\frac{(y^3)^4}{(y^4)^2}$$

Solution: y^4

15. Simplify 1

$$\frac{3\,(x^3y)^2}{(x^2y)^2} \div \frac{12x^4y^2}{(2x^3y)^2}$$

Solution: $\frac{3}{4}$

16. Copy and complete (using index law 3). 6

(a) $(a^6)^{\_\_\_\_} = a^{24}$

Solution: 4

(b) $(b^3)^{\_\_\_\_} = b^{21}$

Solution: 7

(c) $(m^6)\_\_\_\_ = m^{30}$

Solution: 5

(d) $(\_\_\_\_)^6 = p^{36}$

Solution: p^6

(e) $(\_\_\_\_)^4 = a^8$

Solution: a^2

(f) $(\_\_\_\_)^3 = m^{15}$

Solution: m^5

17. 6

(a) Is it true that $(a^2)^6 = (a^6)^2$?

Solution: Yes, both equal a^{12}.

(b) Is it true that $(b^4)^7 = (b^7)^4$?

Solution: Yes, both equal b^{28}.

(c) Generalise your result.

Solution: $(a^m)^n = (a^n)^m = a^{mn}$

18. Simplify by expanding the brackets. [4]

(a) $(xy^3)^2$

Solution: x^2y^6

(b) $(a^2b)^4$

Solution: a^8b^4

(c) $\left(\frac{a}{b}\right)^5$

Solution: $\frac{a^5}{b^5}$

(d) $\left(\frac{x^2}{y}\right)^3$

Solution: $\frac{x^6}{y^3}$

19. Simplify: [2]

(a)

$$\left(\frac{x^3}{y^2}\right)^2 \div \left(\frac{x}{y^2}\right)^3$$

Solution: x^3

(b)

$$\left(\frac{2x^4}{y}\right)^5 \div \left(\frac{4x^3}{y^3}\right)^2$$

Solution: $\frac{32x^8}{y}$

20. Simplify: [2]

(a)

$$\frac{(2x^2y^3)^3 \times (5xy^2)^2}{(10x^2y)^2 \times (xy)^3}$$

Solution: $25y$

(b)

$$\frac{(6ab)^3 \times 2a^7b^4}{(2ab)^4 \times (3a^2b)^2}$$

Solution: $3a^2$

21. Copy and complete. [9]

(a) $(a^2b^3)^4 = a^8b^{12}$ (d) $(x^4y^7)^{-1} = 1$ (g) $(7m^3)^2 = 49m^6$

(b) $(m^5n^4)^6 = m^{30}n^{24}$ (e) $(2a^2)^4 = 16a^8$ (h) $(4\ell^3m)^3 = 64\ell^9m^3$

(c) $(p^3q)^3 = p^9q^3$ (f) $(3q^3)^3 = 27q^9$ (i) $(5m^5n^3)^2 = 25m^{10}n^6$

Negative Indices

1. Express with a positive index and then evaluate. [6]

(a) 3^{-2}

Solution: $\frac{1}{9}$

(d) 5^{-3}

Solution: $\frac{1}{125}$

(b) 6^{-2}

Solution: $\frac{1}{36}$

(e) 3^{-4}

Solution: $\frac{1}{81}$

(c) 9^{-2}

Solution: $\frac{1}{81}$

(f) 10^{-5}

Solution: $\frac{1}{100000}$

2. Write each fraction as a power of a prime with a negative index. [6]

(a) $\frac{1}{27}$

Solution: 3^{-3}

(d) $\frac{1}{125}$

Solution: 5^{-3}

(b) $\frac{1}{49}$

Solution: 7^{-2}

(e) $\frac{1}{169}$

Solution: 13^{-2}

(c) $\frac{1}{121}$

Solution: 11^{-2}

(f) $\frac{1}{81}$

Solution: 3^{-4}

3. Express with positive indices, evaluating where possible. [6]

(a) $3a^{-4}$

Solution: $\frac{3}{a^4}$

(b) $5x^{-7}$

Solution: $\frac{5}{x^7}$

(c) $4a^{-5}$

Solution: $\frac{4}{a^5}$

(d) $\frac{1}{x^{-3}}$

Solution: x^3

(e) $3^{-2}a^{-2}$

Solution: $\frac{1}{9a^2}$

(f) $4^{-2}x^{-2}$

Solution: $\frac{1}{16x^2}$

4. Simplify where possible and then evaluate. 4

(a) $\left(3\frac{1}{3}\right)^{-2}$

Solution: $\frac{1}{11.11}$

(b) $\left(\frac{2}{3}\right)^{-3}$

Solution: $\frac{27}{8}$

(c) $7^3 \times 7^{-5}$

Solution: 7^{-2}

(d) $4^3 \times 4^{-5}$

Solution: 4^{-2}

5. Write as a single power and then evaluate. 6

(a)

$$\frac{3^8}{3^9}$$

Solution: 3^{-1}

(b)

$$\frac{6^5}{6^8}$$

Solution: 6^{-3}

(c)

$$\frac{7^1}{7^3}$$

Solution: 7^{-2}

(d)

$$\frac{5^7}{5^{10}}$$

Solution: 5^{-3}

(e)

$$\frac{3^5}{3^9}$$

Solution: 3^{-4}

(f)

$$\frac{12^{12}}{12^{14}}$$

Solution: 12^{-2}

6. Express with negative index. [3]

(a) $\frac{3}{2x^4}$

Solution: $3 \cdot 2^{-1}x^{-4}$

(b) $\frac{4}{3x^7}$

Solution: $4 \cdot 3^{-1}x^{-7}$

(c) $\frac{2}{3x^5}$

Solution: $2 \cdot 3^{-1}x^{-5}$

7. Evaluate. [7]

(a) $\left(\frac{4}{5}\right)^{-2}$

Solution: $\frac{25}{16}$

(b) $\left(2\frac{1}{4}\right)^{-2}$

Solution: $\frac{16}{81}$

(c) $\left(1\frac{1}{5}\right)^{-3}$

Solution: $\frac{125}{216}$

8. Simplify, expressing the answer with positive indices. [9]

(a) $a^{-3}b^{-5} \times a^5b^{-3}$

Solution: a^2b^{-8}

(b) $3x^{-2}y^5 \times 5x^{-7}y^{-2}$

Solution: $15x^{-9}y^3$

(c) $7a^3m^{-4} \times 8a^{-5}m^{-3}$

Solution: $56a^{-2}m^{-7}$

(d) $3r^2s^3 \times 4r^{-3}s^{-5}$

Solution: $12r^{-1}s^{-2}$

(e) $\frac{16a^{-4}}{8a^5}$

Solution: $2a^{-9}$

(f) $\frac{18a^{-4}}{4a^5}$

Solution: $4.5a^{-9}$

(g) $\frac{27m^{-3}}{9m^{-2}}$

Solution: $3m^{-1}$

(h) $\frac{72a^4b^{-3}}{36ab^{-2}}$

Solution: $2a^3b^{-1}$

(i) $\frac{7a^2b^{-3}c^{-4}}{21a^5b^{-7}c^{-9}}$

Solution: $\frac{1}{3}a^{-3}b^4c^5$

9. Copy and complete. [9]

(a) $9^5 \times 9^{-1} = 9^4$

(b) $b^9 \times b^{-2} = b^7$

(c) $a^{11} \div a^{-3} = a^{14}$

(d) $b^7 \div b^{-8} = b^{15}$

(e) $e^{-7} \div e^2 = e^{-5}$

(f) $(m^{-2})^{-5} = m^{10}$

(g) $(3a^3)^{-3} = \frac{1}{27a^9}$

(h) $(\frac{a^2}{b^3})^{-3} = \frac{a^6}{b^9}$

(i) $(\frac{m^2n^3}{p})^{-6} = \frac{m^{12}n^{18}}{p^6}$

10. Simplify, expressing the answers with positive indices. Evaluate powers where possible. [6]

(a) $\left(5x^4y^6\right)^{-3} \times \left(5^2xy^{-1}\right)^3$

Solution: $x^{-9}y^{-9}$

(b) $\left(5m^2n^{-3}\right)^{-2} \times 2\left(m^{-2}n^3\right)^2$

Solution: $2m^{-2}n^6$

(c) $\dfrac{\left(x^2\right)^2}{y} \times \dfrac{\left(y^2\right)^{-3}}{x^3}$

Solution: xy^{-5}

(d) $\dfrac{\left(2x^3\right)^{-2}}{y^4} \times \dfrac{\left(2x^7\right)^2}{3y^5}$

Solution: $\frac{8x^{10}}{3y^9}$

(e) $\dfrac{\left(m^2n^3\right)^2}{p^{-3}} \times \left(mnp^{-2}\right)^{-3}$

Solution: $m^{-1}n^{-6}p^3$

(f) $\dfrac{\left(a^2\right)^3}{b^3} \div \left(\dfrac{a}{b^2}\right)^{-2}$

Solution: a^7b

(g) $\dfrac{\left(3m^2n^3\right)^{-2}}{p^4} \div \dfrac{p^{-3}}{m}$

Solution: $\frac{m^5}{9n^6p}$

5 Marking

Marker's use only.

SECTION	1	2	3	HW	**Total**
MARKS	$\overline{10}$	$\overline{87}$	$\overline{59}$	$\overline{146}$	$\overline{302}$