Year 9 Mathematics
Topic 7b: Index Laws

PEN Education

2024

Contents

1 Introduction

index indices power root exponent reciprocal sig fig base product

This lesson is a continuation from the last: Topic 7A: Index Laws. Today we will begin by conquering **fractional indices** and then we will finally delete any doubt with regards to **significant figures** and **scientific notation**. Like the previous lesson, there will be a large number of questions (if you consult the table at the bottom you'll see there were 302!) — the reason for this is we want these conversions to become instinctive to you!

2 Fractional Indices

Up until now we have been dealing with *whole* numbers in our powers. We have been manipulating expressions such as a^n where $n \in \mathbb{Z}$. But truthfully, there is no reason why we must confine ourselves to $\mathbb{Z}$, we have the tools to do arithmetic with $\mathbb{Q}$, ______________________________.
Later, if you fall in the black hole of mathematics, you will learn to grasp a to the power of *all real* numbers $\mathbb{R}$, things such as 2^π and later even complex numbers (denoted $\mathbb{C}$) to understand the most beautiful equation in mathematics (by votes):

$$e^{\pi i} = -1$$

But for now we will just understand the *quotient* powers thoroughly.
There are 3 main types:

1. numerator of one: $2^{\frac{1}{3}}$
2. numerator of not one: $2^{\frac{2}{3}}$
3. negative versions of the above cases $\uparrow$

You could just remember that $\sqrt{a} = a^{\frac{1}{2}}$. Or you could *understand* that this is obviously true from the arithmetic laws you learnt last lesson.
Recall that $(a^2)^2 = a^4$, then $a^1 = (a^{\frac{1}{2}})^2$ which we could say is the same as $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$. But then which mathematical entity becomes itself when squared?
Thus, obviously $a^{\frac{1}{2}} =$ _______________

2.1 Examples:

1. 3

   (a) $100^{\frac{1}{2}} =$ _______________ (b) $4^{\frac{1}{2}} =$ _______________ (c) $256^{\frac{1}{2}} =$ _______________

We can extend this more generally, where the denominator does not need to be 2. Recall that $\sqrt[3]{8} =$ _______________.

And so $8^{\frac{1}{3}} = 2$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$.

2.2 Examples

1. 6

   (a) $\sqrt[3]{27} =$ _______________ (c) $\sqrt[3]{125} =$ _______________ (b) $27^{\frac{1}{3}} =$ _______________

   (b) $\sqrt[4]{16} =$ _______________ (a) $9^{\frac{1}{2}} =$ _______________ (c) $16^{\frac{1}{4}} =$ _______________

Notice that up until now all of the numerators have been the number 1. It is easy to equip ourselves to handle bigger numbers though. Just recall the law from last lesson:

$$(a^m)^n = a^m n$$

Which we can apply to the $a^{\frac{1}{n}}$'s we have by raising this to whatever number we need. Thus $a^{\frac{2}{3}} = (a^{\frac{1}{3}})^2$.
We can even change the order around to be $(a^2)^{\frac{1}{3}}$ which then looks nicer as $\sqrt[3]{a^2}$.

Positive fractional indices

$$a^{\frac{p}{q}} = (a^{\frac{1}{q}})^p = (\sqrt[q]{a})^p$$

2.3 Examples:

1. 3

   (a) $4^{\frac{3}{2}} =$ ____________ (b) $8^{\frac{2}{3}} =$ ____________ (c) $81^{\frac{3}{4}} =$ ____________

The final thing in this section to learn is the *negative fractional indices*. Recall that

$$a^{-m} = \frac{1}{a^m}$$

.
Then in exactly the same fashion,

$$a^{-\frac{p}{q}} = \frac{1}{a^{\frac{p}{q}}}$$

2.4 Examples:

1. 2

   (a) $4^{-\frac{3}{2}} =$ (b) $(\frac{8}{27})^{-\frac{1}{3}} =$

2.5 Exercises:

1. Evaluate: 3

(a) $\sqrt[3]{8}$ (b) $\sqrt[5]{32}$ (c) $\sqrt[3]{216}$

2. Write using fractional indices. Evaluate, correct to 4 decimal places. 3

(a) $\sqrt{14}$ (b) $\sqrt[4]{64}$ (c) $\sqrt[5]{7}$

3. Evaluate: 6

(a) $4^{\frac{1}{2}}$ (c) $64^{\frac{1}{2}}$ (e) $32^{\frac{1}{5}}$

(b) $27^{\frac{1}{3}}$ (d) $25^{\frac{1}{2}}$ (f) $625^{\frac{1}{4}}$

4. Evaluate: 6

(a) $4^{\frac{5}{2}}$ (c) $32^{\frac{2}{5}}$ (e) $(\sqrt[4]{16})^3$

(b) $25^{\frac{3}{2}}$ (d) $81^{\frac{3}{4}}$ (f) $(\sqrt[3]{27})^2$

5. Simplify: 8

(a) $\left(a^{\frac{1}{2}}\right)^2$

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(b) $\left(b^{\frac{1}{3}}\right)^6$

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(c) $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$

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(d) $y^{\frac{1}{3}} \times y^{\frac{2}{3}}$

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(e) $x^{\frac{3}{2}} \div x^{\frac{1}{2}}$

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(f) $y^{\frac{2}{3}} \div y^{\frac{1}{3}}$

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(g) $(4m^6)^{\frac{1}{2}}$

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(h) $(27n^{12})^{\frac{1}{3}}$

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6. Evaluate: $\boxed{4}$

(a) $4^{-\frac{1}{2}}$

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(b) $25^{-\frac{1}{2}}$

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(c) $\left(\frac{1}{81}\right)^{-\frac{1}{4}}$

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(d) $81^{-\frac{1}{4}}$

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7. Simplify, expressing the answer with positive indices. $\boxed{6}$

(a) $\left(a^{\frac{1}{2}}\right)^{-2}$

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(c) $y^{\frac{1}{3}} \times y^{-\frac{2}{3}}$

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(e) $p^{\frac{3}{4}} \div p^{-\frac{2}{5}}$

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(b) $\left(b^{-\frac{2}{3}}\right)^{6}$

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(d) $p^{\frac{3}{4}} \times p^{-\frac{2}{5}}$

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(f) $q^{\frac{3}{2}} \div q^{-\frac{2}{3}}$

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3 Scientific Notation

This part is far more straight forward than the last. Basically if you have a really small number like 0.000000000001, instead of writing it out this tediously we can just count the zeros and write 1×10——————.
Similarly, if you have a really large number 900000000000000000. You could just write $9 \times 10^{1}7$. It is more concise and less prone to errors.

Definition 1

Tedious:

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Definition 2

Concise:

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Definition 3

Prone:

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3.1 Examples:

1. Write in scientific notation. 8

(a) $610 =$ ____________

(b) $21000 =$ ____________

(c) $0.0067 =$ ____________

(d) $0.00002 =$ ____________

(e) $46000000 =$ ____________

(f) $81 =$ ____________

(g) $0.07 =$ ____________

(h) $8.17 =$ ____________

2. Now go the other way; write the following in decimal form: $\boxed{4}$

(a) $2.1 \times 10^3 =$ ____________

(b) $6.3 \times 10^5 =$ ____________

(c) $5 \times 10^{-4} =$ ____________

(d) $8.12 \times 10^{-2} =$ ____________

3. Simplify and write in scientific notation. $\boxed{4}$

(a) $(3 \times 10^4) \times (2 \times 10^6)$

(b) $(9 \times 10^7) \div (3 \times 10^4)$

(c) $(4.1 \times 10^4)^2$

(d) $(2 \times 10^5)^{-2}$

3.2 Exercises:

1. Write as a power of 10. $\boxed{6}$

(a) $\frac{1}{10} =$ ____________

(b) $\frac{1}{100} =$ ____________

(c) $\frac{1}{1000} =$ ____________

(d) 1 trillionth $=$ ____________

(e) $\frac{1}{100000} =$ ____________

(f) 1 millionth $=$ ____________

2. Write in scientific notation. $\boxed{8}$

(a) $510 =$ ____________

(b) $5300 =$ ____________

(c) $796000000 =$ ____________

(d) $576000000000 =$ ____________

(e) $0.008 =$ ____________

(f) $0.06 =$ ____________

(g) $0.000041 =$ ____________

(h) $0.000000006 =$ ____________

3. Write in decimal form: $\boxed{8}$

(a) $3.24 \times 10^4 =$ ____________ (e) $5.6 \times 10^{-2} =$ ____________

(b) $7.2 \times 10^3 =$ ____________ (f) $1.7 \times 10^{-3} =$ ____________

(c) $2.7 \times 10^6 =$ ____________ (g) $2.01 \times 10^{-3} =$ ____________

(d) $5.1 \times 10^0 =$ ____________ (h) $9.7 \times 10^{-1} =$ ____________

4. Light travels approximately 299000 km in a second. Express this in scientific notation. [1]

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5. The mass of a copper sample is 0.0089 kg. Express this in scientific notation. [2]

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6. The distance between interconnecting lines on a silicon chip for a computer is approximately 0.00000004 m. Express this in scientific notation. [2]

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7. Simplify, expressing the answer in scientific notation. [8]

(a) $(4 \times 10^5) \times (2 \times 10^6)$

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(b) $(2.1 \times 10^6) \times (3 \times 10^7)$

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(c) $(4 \times 10^2) \times (5 \times 10^{-7})$

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(d) $(3 \times 10^6) \times (8 \times 10^{-3})$

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(e) $(5 \times 10^4) \div (2 \times 10^3)$

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(f) $(8 \times 10^9) \div (4 \times 10^3)$

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(g) $(6 \times 10^{-4}) \div (8 \times 10^{-5})$

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(h) $(1.2 \times 10^6) \div (4 \times 10^7)$

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8. If the average distance from the Earth to the Sun is 1.4951×10^8 km and light travels at 3×10^5 km/s, how long does it take light to travel from the Sun to the Earth? $\boxed{3}$

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9. The furthest galaxy detected by optical telescopes is approximately 4.6×10^9 light years from us. How far is this in kilometres? (Light travels at 3×10^5 km/s.) $\boxed{3}$

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4 Significant Figures

This is the final section and it relies on an understanding of the previous section. Conceptually, it is almost obvious what a *significant* figure is - it is a figure which is important.

Let us consider 1.618. This number has 4 significant figures. We could equally write 1.618000000 . . ., but these zeros do not add any accuracy or information to our number. As such they are considered insignificant.

Similarly, there are ____________ significant figures in 5.9736 and 8 significant figures in ____________.

Worked Example:

$0.00034061 = 3.4061 \times 10^{-4}$

$$\begin{aligned} &\approx 3 \times 10^{-4} \quad (\text{ correct to 1 significant figure }) \\ &\approx 3.4 \times 10^{-4} \quad (\text{ correct to 2 significant figures }) \\ &\approx 3.41 \times 10^{-4} \quad (\text{ correct to 3 significant figures }) \\ &\approx 3.406 \times 10^{-4} \quad (\text{ correct to 4 significant figures }) \end{aligned}$$

4.1 Examples:

1. Write in scientific notation and then round correct to 3 significant figures. $\boxed{2}$

   (a) $235.674 =$ ____________ (b) $0.00724546 =$ ____________

2. Write in scientific notation and then round correct to 2 significant figures. $\boxed{2}$

   (a) $276000000 \approx$ ____________ (b) $0.000000654 \approx$ ____________

4.2 Exercises:

1. Write in scientific notation, correct to 3 significant figures. $\boxed{6}$

   (a) 2.7043

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   (b) 634.96

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   (c) 8764.37

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   (d) 256412

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   (e) 0.003612

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   ....

   (f) 0.024186

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   ....
   ....

2. $\boxed{16}$

	4 sig. figs	3 sig. figs	2 sig. figs	1 sig. fig.
274.62				
0.041236				
1704.28				
1.9925×10^{27}				

5 Homework

5.1 Fractional Indices

1. Evaluate: $\boxed{3}$

   (a) $\sqrt[4]{81}$ (b) $\sqrt[3]{64}$ (c) $\sqrt[5]{2^{10}}$

2. Write using fractional indices. Evaluate, correct to 4 decimal places. $\boxed{2}$

   (a) $\sqrt[7]{11}$ (b) $\sqrt[3]{2^7}$

3. Evaluate: $\boxed{6}$

   (a) $243^{\frac{1}{5}}$

   (b) $81^{\frac{1}{4}}$

   (c) $125^{\frac{1}{3}}$

   (d) $64^{\frac{1}{3}}$

   (e) $216^{\frac{1}{3}}$

   (f) $49^{\frac{1}{2}}$

4. Evaluate: $\boxed{6}$

   (a) $125^{\frac{2}{3}}$

   (b) $64^{\frac{5}{6}}$

   (c) $216^{\frac{2}{3}}$

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(d) $243^{\overline{5}}$

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(e) $\sqrt[5]{32^4}$

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(f) $\sqrt[3]{2^6}$

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5. Simplify: $\boxed{8}$

(a) $(c^{12})^{\frac{1}{4}}$

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(b) $(c^{10})^{\frac{1}{5}}$

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(c) $p^{\frac{3}{4}} \times p^{\frac{2}{5}}$

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(d) $q^{\frac{3}{2}} \times q^{\frac{2}{3}}$

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(e) $p^{\frac{3}{4}} \div p^{\frac{2}{5}}$

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(f) $q^{\frac{3}{2}} \div q^{\frac{2}{3}}$

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(g) $\left(2x^{\frac{2}{3}}\right)^3$

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(h) $\left(3y^{\frac{1}{2}}\right)^4$

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6. Evaluate: $\boxed{4}$

(a) $\left(\frac{64}{27}\right)^{-\frac{1}{3}}$

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(b) $32^{-\frac{2}{5}}$

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(c) $\left(\frac{16}{81}\right)^{-\frac{1}{4}}$

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(d) $\left(\frac{32}{243}\right)^{-\frac{1}{5}}$

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7. Simplify, expressing the answer with positive indices. $\boxed{6}$

(a) $\left(3y^{\frac{1}{2}}\right)^{-4}$

. . . .

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(b) $x^{\frac{1}{2}} \times x^{-\frac{3}{2}}$

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. . . .

(c) $x^{\frac{3}{2}} \div x^{-\frac{1}{2}}$

. . . .

. . . .

. . . .

(d) $y^{\frac{2}{3}} \div y^{-\frac{1}{3}}$

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. . . .

(e) $(27n^{-12})^{\frac{1}{3}}$

. . . .

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. . . .

(f) $\left(2x^{-\frac{2}{5}}\right)^{5}$

5.2 Scientific Notation

1. Write in scientific notation. [4]

(a) 26000

(b) 4000000000000

(c) 0.00072

(d) 0.000000206

2. Write in decimal form: [4]

(a) 8.6×10^2

(b) 7.2×10^1

(c) 8.72×10^{-4}

(d) 2.6×10^{-7}

3. Simplify, expressing the answer in scientific notation. [6]

(a) $(4 \times 10^{-2})^2 \times (5 \times 10^7)$

(b) $(6 \times 10^{-3}) \times (4 \times 10^7)^2$

(c) $\frac{(4\times10^5)^3}{(8\times10^4)^2}$

. . . .

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(d) $\frac{(2\times10^{-1})^5}{(4\times10^{-2})^3}$

. . . .

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. . . .

4. If light travels at 3×10^5 km/s and our galaxy is approximately 80000 light years across, how many kilometres is it across? (A light year is the distance light travels in a year.) $\boxed{2}$

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5. The mass of a hydrogen atom is approximately 1.674×10^{-27} kg and the mass of an electron is approximately 9.1×10^{-31} kg. How many electrons, correct to the nearest whole number, will have the same mass as a single hydrogen atom? $\boxed{2}$

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6. In a lottery there are $\frac{45\times44\times43\times42\times41\times40}{720}$ different possible outcomes. If I mark each outcome on an entry form one at a time, and it takes me an average of 1 minute to mark each outcome, how long will it take me to cover all different possible outcomes? $\boxed{2}$

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5.3 Significant Figures

1. Write in scientific notation, correct to 2 significant figures. $\boxed{4}$

(a) 368.2

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(b) 278000

(c) 0.004321

(d) 0.000021906

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2. Use a calculator to evaluate the following, giving the answer in scientific notation correct to 3 significant figures. $\boxed{4}$

(a) 3.24×0.067

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. . . .
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(b) $4.736 \times 10^{13} \times 2.34 \times 10^{-6}$

. . . .
. . . .
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(c) $0.0276^2 \times \sqrt{0.723}$

. . . .
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(d) $\frac{6.54(5.26^2+3.24)}{5.4+\sqrt{6.34}}$

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3. Use a calculator to evaluate, giving the answer in scientific notation correct to 4 significant figures. $\boxed{4}$

(a) 1.234×0.1988

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(b) $1.234 \div 0.1988$

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(c) 1.9346^3

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(d) $(7.919 \times 10^{21})^2$

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6 Marking

Marker's use only.

SECTION	1	2	3	4	HW	**Total**
MARKS	$\overline{}$ 0	$\overline{}$ 50	$\overline{}$ 57	$\overline{}$ 26	$\overline{}$ 67	$\overline{}$ 200