Year 9 Mathematics | Topic 1 | Algebra Revision

PEN Education

October 20, 2023

. . . .

Contents

. . . .

1 Introduction

What does the word *algebra* mean?

. . . .

. . . .

. . . .

. . . .

Where does the word **algebra** come from?

. . . .

. . . .

. . . .

What is an example of *algebra*?

. . . .

. . . .

What can I do with ALGEBRA?

. . . .

. . . .

. . . .

2 Substitution

What is this?

. . . .

. . . .

. . . .

Definition 1

Pronumeral := . . . .

Definition 2

Numerical Value := . . . .

What do we need to be careful of?

. . . .

. . . .

. . . .

2.1 Examples

(a) Evaluate $2x$ when $x = 3$

. . . .

. . . .

(b) Evaluate $5a + 2b$ when $a = 2$ and $b = -3$

. . . .

. . . .

(c) Evaluate $2p(3p - 2)$ when $p = 1$ and $q = -2$

. . . .

. . . .

(d) Evaluate $7m - 4n$ when $m = -3$ and $n = -2$

. . . .

. . . .

(e) Evaluate $a + 2b - 3c$ when $a = 3, b = -5, c = -2$

. . . .

. . . .

2.2 Exercises

1. Evaluate $2x - 3y$ when:

   (a) $x = \frac{2}{5}, y = -\frac{1}{4}$

   (b) $x = \frac{1}{3}, y = \frac{1}{6}$

2. Evaluate $p^2 - 2q$ when:

   (a) $p = -7, q = 2$

   (b) $p = -\frac{1}{3}, q = \frac{5}{6}$

3 Like Terms

Why should we group like terms?

Give an example of grouping like terms.

3.1 Examples

Which of the following are pairs of like terms?

(a) $3x, 2x$

(b) $3m, 2c$

(c) $3x^2, 3x$

(d) $2x^2y, 3yx^2$

(e) $2mn, 3nm$

(f) $5y^2, 6y^2x$

Simplify each expression if possible:

(a) $4a + 7a =$ ....

(b) $3x^2y + 4x^2 - 2x^2y =$ ....

(c) $5m + 6n =$ ....

(d) $9b + 2c - 3b + 6c =$ ....

(e) $3z + 5yx - z - 6xy =$ ....

(f) $6x^3 - 4x^2 + 5x^3$ ....

3.2 Exercises

1. Simplify:

   (a) $\frac{x}{2} + \frac{x}{3}$

   ....

   ....

   ....

   (b) $\frac{3x}{4} - \frac{2x}{5}$

   ....

   ....

   ....

2. Which of the following are pairs of like terms?

   (a) $12m, 5m$

   (b) $-6a, 7b$

   (c) $6ab, -7b$

   (d) $6x^2, -7x^2$

3. Simplify each expression by collecting like terms.

   (a) $8b + 3b$ ....

   (b) $6x^2 + 4x^2$ ....

   (c) $7f - 3f + 9f$ ....

4. Fill in the missing term.

   (a) $8mn+$ .... $= 12mn$

   (b) $6m^2-$ .... $= m^2$

   (c) $-7a^2b+$ .... $= a^2b$

5. Simplify by collecting like terms.

   (a) $8p + 6 + 3p - 2 =$ ....
   (b) $10ab + 11b - 12b + 3ab =$ ....
   (c) $4p^2 - 3p - 8p - 3p^2 =$ ....
   (d) $-4x^2 + 3x^2 - 3y - 7y =$ ....
   (e) $7x^3 + 6x^2 - 4y^3 - x^2 =$ ....
   (f) $-3ab^2 + 4a^2b - 5ab^2 + a^2b =$ ....

6. Simplify:

   (a) $\frac{c}{6} + \frac{c}{7}$

   ....

   ....

   (b) $\frac{x}{7} - \frac{x}{8}$

   ....

   ....

   (c) $c - \frac{c}{7}$

   ....

   ....

   (d) $\frac{2x}{3} + \frac{x}{4}$

   ....

   ....

   (e) $\frac{5x}{3} + \frac{x}{2}$

   ....

   ....

   (f) $\frac{5x}{11} - \frac{2x}{3}$

   ....

   ....

4 Multiplication and Division

This part is interesting. In the last section we saw that we could **not** further simplify terms that were different such as $2x + 4y$. But now with multiplication and division we can! $2x \times 4y = 8xy$ and $2x \div 4y = \frac{\cancel{2}x}{\cancel{4}y} = \frac{x}{2y}$. Isn't that cool?
Okay, now you guys try:

4.1 Examples

1. Multiplications

   (a) $4 \times 3a =$ ....
   (b) $2d \times 5e =$ ....
   (c) $4m \times 5m =$ ....
   (d) $3p \times 2pq =$ ....
   (e) $3x \times (-6) =$ ....
   (f) $-5ab \times -3bc$ ....

2. Divisions

   (a) $24x \div 6 =$ ....
   (b) $36a \div 4$ ....
   (c) $-18x^2 \div (-3)$ ....
   (d) $\frac{15a}{3}$ ....
   (e) $\frac{12x}{21}$ ....
   (f) $\frac{-24xy}{6y}$ ....

4.2 Exercises

1. Rewrite as a single fraction:

   (a) $\frac{2a}{5} \times \frac{a}{4} = \ldots\ldots\ldots\ldots\ldots$ (c) $\frac{4p}{q} \times \frac{3}{2p} = \ldots\ldots\ldots\ldots$ (e) $\frac{2x}{3} \div \frac{3x}{5} = \ldots\ldots\ldots\ldots$

   (b) $\frac{3x}{7} \times \frac{5y}{12} = \ldots\ldots\ldots\ldots$ (d) $\frac{15}{x} \times \frac{2}{3x} = \ldots\ldots\ldots\ldots$ (f) $\frac{6a}{7b} \div \frac{2ab}{3} = \ldots\ldots\ldots\ldots$

2. Simplify

   (a) $5c \times 2d = \ldots\ldots\ldots\ldots$ (c) $-2m \times (-4m) = \ldots\ldots$ (e) $7 \times 15p \div 21 = \ldots\ldots\ldots$

   (b) $-6l \times (-5m) = \ldots\ldots$ (d) $24a^2 \div 8 = \ldots\ldots\ldots\ldots$ (f) $18y \div 6 \times 2 = \ldots\ldots\ldots$

3. Simplify by first cancelling out common factors:

   (a) $\frac{14p}{21} = \ldots\ldots\ldots$ (c) $\frac{2xy}{6xy} = \ldots\ldots\ldots$ (e) $\frac{2y}{5} \times \frac{y}{4} = \ldots\ldots$ (g) $\frac{2yz}{5xy} \times \frac{3xy}{4yz} = \ldots$

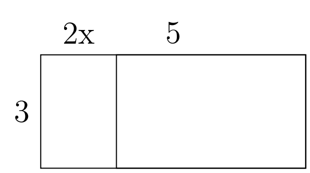
   (b) $\frac{22x^2}{33} = \ldots\ldots\ldots$ (d) $\frac{-4xy}{8x} = \ldots\ldots\ldots$ (f) $\frac{p}{6q} \times \frac{9p}{4q} = \ldots\ldots$ (h) $\frac{2y}{5} \div \frac{y}{4} = \ldots\ldots$

5 Simple Expansion of Brackets

Often times algebraic concepts have a geometric meaning too. You've now done enough arithmetic with pronumerals to be able to learn this secret of the universe.
Consider $3(2x+5)$. You can expand this by distributing the 3 to each term in the brackets like so:

$$(a+4)(b+3) = \tag{1}$$

Or you can think about this as having some kind of original rectangle with dimensions 3 by $2x$ and then extending the width by 5.



Now finding the area of the enlarged shape is algebraically equivalent to $3(2x+5)$ and often times expanding this will make substitution easier if you know what the value of x is!
These kinds of expansions are the backbone of mathematics and becoming proficient at these will help you simplify harder problems. Let's get better at expanding:

5.1 Examples

1. Expand:

   (a) $2(a+3) = \ldots\ldots\ldots\ldots$ (b) $3(x-2) = \ldots\ldots\ldots\ldots$ (c) $4(2m-7) = \ldots\ldots\ldots$

2. Now try:

   (a) $5(a+1)+6 = \ldots\ldots\ldots$ (b) $4(2b-1)+7 = \ldots\ldots\ldots$ (c) $6(d+5)-3d = \ldots\ldots$

3. Can you handle some more terms?

   (a) $2(b+5)+3(b+2) = \ldots\ldots\ldots\ldots\ldots\ldots$ (b) $3(x-2)-2(x+1) = \ldots\ldots\ldots\ldots\ldots$

5.2 Exercises

Have a go at these ones yourselves:

1. $\frac{3}{5}(6x+\frac{7}{3}) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
2. $\frac{4}{3}(6x+11)+\frac{2}{3} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$
3. $-12(4y-5) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
4. $\frac{2}{3}(12p+6) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
5. $-\frac{1}{2}(10d-6) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
6. $-\frac{4}{5}(25m-100) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$
7. $\frac{3}{5}(\frac{x}{6}+\frac{1}{3}) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
8. $-\frac{3}{5}(\frac{a}{3}-\frac{2}{3}) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
9. $c(c-5) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
10. $2i(5i+7) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

6 Binomial Products

Welcome to some respectable mathematics. Binomals look like this: (x+something)(y+something else). We are going to learn how to expand any variant of these, and then we will look at the special cases when x and y are the same and the *something*'s are also the same; i.e. $(x+a)(x+a)$. (There is a quick trick for solving these). Then we shall conclude the class with the second special case of the binomials - *The Difference of Two Squares*. They come in the shape of $(x+a)(x-a)$, and also can be easily expanded with a trick!
Before we get stuck in to the expansion tricks, let's make sure we understand what we are expanding.

What does the prefix ***bi*** mean?

....

....

Examples of ‘**bi**’ things include ....
.
Thus a **bi**nomial means ....
.
Now let us expand $(a+2)(b+5)$. You just need to distribute each term in the first brackets with every term of the next set of brackets.

$$(a+2)(b+5) = ab + 2b + 5a + 10 \tag{2}$$

If at first you are struggling to remember the steps, just remember the acronym **FOIL**, **F**irst **O**utside **I**nside **L**ast.
Once again this has a geometric interpretation:

	a	2
b	ab	$2b$
5	$5a$	10

And the area can now be computed by adding all the parts: $ab + 2b + 5a + 10$, which is what our algebraic expansion told us too!

6.1 Examples

1. Expand the following:

   (a) $(x+4)(x+5) =$

   ....

   ....

   (b) $(x+3)(x-2) =$

   ....

   ....

   (c) $(x-4)(x-3) =$

   ....

   ....

   (d) $(2y+1)(3y-4) =$

   ....

   ....

6.2 Exercises

(a) $(a+3)(a+9) =$

....

....

(b) $(a+8)(9+a) =$

....

....

(c) $(p-6)(p+4) =$

....

....

(d) $(x+3)(x-8) =$

....

....

(e) $(x+7)(x-4) =$

....

....

(f) $(5x+1)(x+2) =$

....

....

(g) $(4m+3)(2m-1) =$

....

....

(h) $(2x-7)(3x-1) =$

....

....

(i) $(2b+3)(4b-2) =$

....

....

(j) $(4c+d)(2c-3d) =$

....

....

(k) $(3x-y)(2x+5y) =$

....

....

(l) $(2p-5q)(3q-2p) =$

....

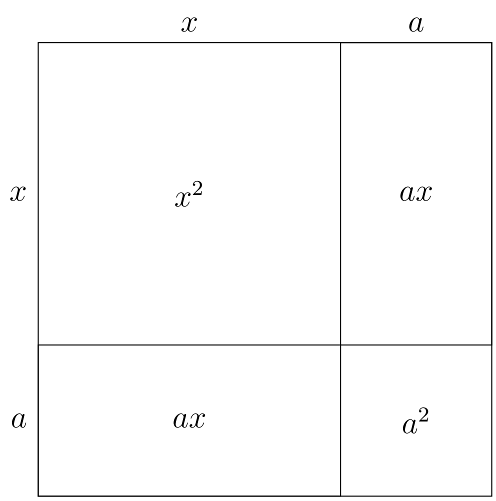
....

7 Perfect Squares

This is one of the special cases mentioned earlier. Our general binomial looks like $(x+a)(y+b)$, but perfect squares are easier and look like $(x+a)(x+a)$ which can then be simplified to be $(x+a)^2$.
Remember the trick mentioned earlier? This is it:

1. Take the first term, square it
2. Take the last term, square it
3. Multiply all the terms with each other

Thus we have $(x+a)^2 = x^2 + 2ax + a^2$. Simple as that.
Here is the geometric intuition:



We take a square of x units and extend it to a square of length $x+a$:

7.1 Examples

Let's only do a few examples this time. We'll come back and do more practise after covering *differences of two squares.*

1. $(x-5)^2 =$ ....
2. $(x+7)^2 =$ ....
3. $(3x-1)^2 =$ ....

8 Difference of Two Squares

We shall cover this one quickly so you have time to do a brick of exercises after :D. Difference of Two Squares are the second special case of the **binomial** expansion, and come in the form $(x+a)(x-a)$. Expanding this out with our usual **FOIL** method gives $x^2 + \cancel{ax} - \cancel{ax} - a^2$ which just leaves $x^2 - a^2$; how convenient!
This time I will leave the geometric intuition as an exercise, feel free to come to me before next class to explain your ideas!

8.1 Exercises

1. Let's Practise:

   (a) $(x-5)(x+5) =$

   ....

   ....

   (b) $(3x-4)(3x+4) =$

   ....

   ....

   (c) $(a+b)(a-b) =$

   ....

   ....

2. Now back to perfect squares:

   (a) $(x+1)^2 =$

   ....

   ....

   (b) $(x+5)^2 =$

   ....

   ....

   (c) $(2+x)^2) =$

   ....

   ....

   (d) $(x+20)^2) =$

   ....

   ....

3. Try a mix now:

(a) $(3x-2)(3x+2) =$

....

....

....

(b) $(3a-4b)^2 =$

....

....

(c) $(2x+3y)^2) =$

....

....

(d) $(5a+2b)(5a-2b)) =$

....

....

(e) $(\frac{x}{2}+3)^2) =$

....

....

(f) $(3c-b)^2) =$

....

....

9 Homework

Please attempt every question in your exercise books!

1. Evaluate $2m(m-3n)$ when:

   (a) $m = 3, n = 5$ (b) $m = -3, n = -2$ (c) $m = \frac{1}{3}, n = \frac{1}{2}$

2. Evaluate $\frac{p+2q}{3r}$ when $p = 7, q = -2, r = 2$

3. Evaluate $\frac{x+y}{3}$ when $x = -6, y = -5$

4. Fill in the missing term:

   (a) $2a+$.............. $= 7a$ (b) $5m^2-$ .... $= -6m^2n$ (c) $-6lm+$.......... $= lm$

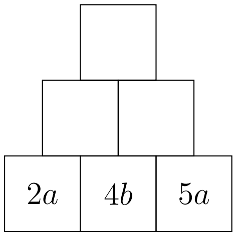
5. Simplify by collecting like terms:

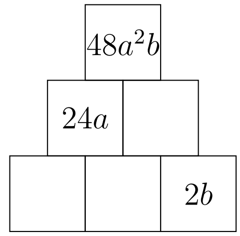
   (a) $9a^2 + 5a^2 - 12a^2$
   (b) $14a^2d - 10a^2d - 6a^2d$
   (c) $17m^2 - 14m^2 + 8m^2$
   (d) $-4x^2 + 3x^2 - 3y - 7y$
   (e) $7x^3 + 6x^2 - 4y^3 - x^2$
   (f) $-3ab^2 + 4a^2b - 5ab^2 + a^2b$

6. Simplify

   (a) $4a \times 3b$
   (b) $-2p \times (-3q)$
   (c) $27y \div 3$
   (d) $3 \times 12t \div 9$
   (e) $24x \div 8 \times 3$
   (f) $-\frac{12m}{18}$
   (g) $\frac{12ab}{4a}$
   (h) $\frac{3x}{5} \times \frac{2}{3}$
   (i) $\frac{2}{5a} \times \frac{1}{4a}$
   (j) $\frac{3x}{5} \div \frac{3}{4}$
   (k) $\frac{9y}{2} \div 18$
   (l) $\frac{5p}{6} \div (-\frac{10p}{3})$

7. Fill in the missing boxes. Each box contains the product of the 2 boxes below it.

(a) 

(b) 

8. Expand:

   (a) $b(b+7)$ (c) $-k(5k-4)$ (e) $4c(2c-d)$ (g) $3p(2-5pq)$
   (b) $4h(5h-7)$ (d) $-4x(3x-5)$ (f) $-3x(2x+5y)$ (h) $-10b(3a-7b)$

9. Expand and collect like terms

   (a) $\frac{1}{4}(x+2)+\frac{x}{3}$ (c) $-\frac{1}{2}(3x+2)-\frac{2x}{5}$ (e) $2p(3p+1)-4(2p+1)$
   (b) $\frac{3}{7}(3x+5)+\frac{x}{3}$ (d) $2p(3p+1)-5(p+1)$ (f) $4z(4z-2)-z(z+2)$

10. Expand:

    (a) $(x-6)(x-4)$ (c) $(4x+3)(2x-1)$ (e) $(x+3)(x+3)$ (g) $(2x+3)(2x+3)$
    (b) $(4x+1)(3x-1)$ (d) $(x-4)(2x+5)$ (f) $(2x-5)(x+3)$ (h) $(\frac{2b}{3}+2)(\frac{b}{5}-2)$

11. Fill in the blanks:

    (a) $(x+5)(\ldots\ldots\ldots\ldots\ldots)=x^2+8x+15$ (d) $(x+\ldots\ldots.)(x+6)=x^2+9x+\ldots\ldots.$
    (b) $(x+3)(\ldots\ldots\ldots\ldots\ldots)=x^2-2x-15$ (e) $(2x+3)(\ldots\ldots.)=2x^2+7x+\ldots\ldots.$
    (c) $(3x+4)(\ldots\ldots\ldots\ldots\ldots)=3x^2+x-4$ (f) $(\ldots.x-3)(\ldots.x5\ldots.)=12x^2-x-6$

12. Expand

    (a) $(x-7)^2$ (b) $(a+8)^8$ (c) $(9+x)^2$ (d) $x-11)^2$

13. Expand

    (a) $(\frac{2x}{5}-1)^2$ (b) $(\frac{3x}{4}+\frac{2}{3})^2$

14. Evaluate the following using $(a+b)^2=a^2+2ab+b^2$ and $(a-b)^2=a^2-2ab+b^2$.

    (a) $(1.01)^2$ (b) $(0.99)^2$ (c) $(4.01)^2$

15. Expand and collect like terms

    (a) $(x-2)^2+(x-4)^2$ (c) $x^2+(x+1)^2+(x+2)^2+(x+3)^2$
    (b) $(2x+5)^2+(2x-5)^2$ (d) $(\frac{x}{2}+1)^2+(\frac{x}{2}-1)^2$

16. Expand

(a) $(z-7)(z+7)$

(b) $(10-x)(10+x)$

(c) $(3x-2)(3x+2)$

(d) $(\frac{x}{2}+3)(\frac{x}{2}-3)$

(e) $(\frac{x}{3}+\frac{1}{2})(\frac{x}{3}-\frac{1}{2})$

(f) Is a^2-2a+1 a perfect square expansion or a difference of 2 squares?

9.1 Challenge Problems

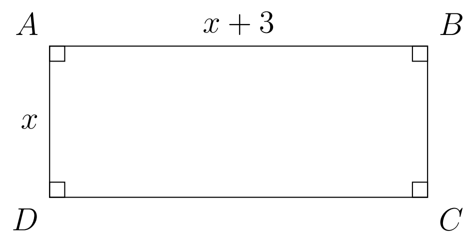
1. (a) Show that the perimeter of the rectangle is $4x+6$cm

   (b) Find the perimeter if $AD=2$cm

   (c) Find x if the perimeter $=36$cm

   (d) Find the area of $ABCD$ in terms of x

   (e) Find the area of the rectangle if $AB=6$cm



2. Expand and collect:

   (a) $(x-1)(x^2+x+1)$

   (b) $(x-1)(x^4+x^3+x^2+x+1)$

   (c) What do you expect the result of expanding $(x-1)(x^9+x^8+\cdots+1)$ will be?

10 The End