Year 9 Mathematics | Topic 2 | Pythagoras and Surds [1/2]

PEN Education

2024

Contents

1 Introduction

In today's lesson we are going to begin by reviewing **Pythagoras' Theorem** which you should have been introduced to in Year 8, and then delve deeper. We will examine how "*irrational*" numbers pop out, what the significance of such numbers are, and what the converse of this theorem states.
We will also learn the arithmetic of surds: i.e. what can be done with $\sqrt{8} + 2\sqrt{2}$ and $\sqrt{3} \div \sqrt{6}$.
Next week, we shall tackle more complex and less trivial problems of *rationalising the denominator* and *3D trigonometry*!

2 The Pythagorean Theorem

Let us begin with a Theorem:

Theorem

For a right-angled triangle, the following relationship holds between the three sides:

$$a^2 + b^2 = c^2$$

Where c is the length of the hypotenuse, and a and b, the lengths of the remaining two sides.

Definition 1

2.0.1 Theorem:

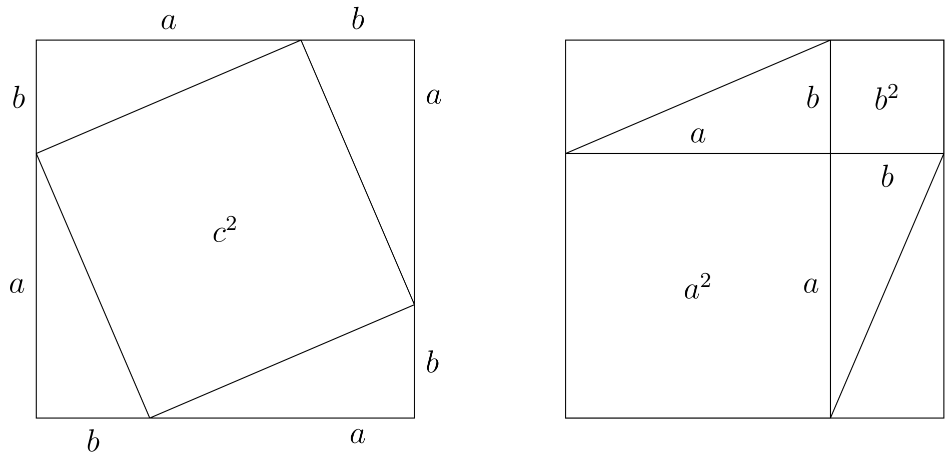
Solution: A statement that has been proven on the basis of previously established statements.

Definition 2

2.0.2 Hypotenuse

Solution: The longest side of a right-angled triangle, opposite the right angle.

This is probably the first **Theorem** that you have ever seen and it is also one of the most mathematically significant ones. It must be emphasised that a Theorem means nothing without a **Proof**. 12-year old Einstein, President James Garfield, and Euclid all found unique proofs to $a^2 + b^2 = c^2$. Below we outline Pythagoras' original proof by rearrangement so that you can visually grasp the geometry of this fact.



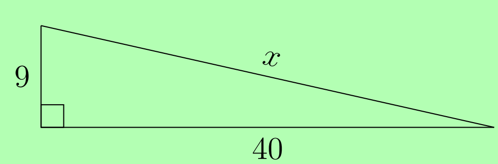
Explanation

If you don't understand it immediately, do not panic. It takes a moment to click. What is happening here is that the 4 triangles in the first figure are the same as the 4 triangles in the second figure. So the free space of figure 1 must be the same as the free space in figure 2, and thus $c^2 = a^2 + b^2$.

Now that we all believe *The Pythagorean Theorem*, let us use this mathematical result to discover things about our world:

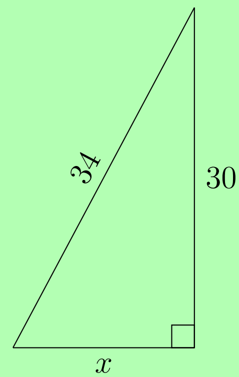
3 Examples

1. 1



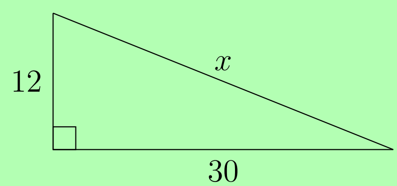
Solution: $x = \sqrt{40^2 + 9^2} = \sqrt{1600 + 81} = \sqrt{1681} = 41$

2. [1]



Solution: $x = \sqrt{34^2 - 30^2} = \sqrt{1156 - 900} = \sqrt{256} = 16$

3. [1]



Solution: $x = \sqrt{30^2 + 12^2} = \sqrt{900 + 144} = \sqrt{1044} \approx 32.31$

What is the difference between the third example and the previous 2?

Solution: The third example involves finding the hypotenuse x given the two shorter sides, whereas the previous two examples involve finding on of the shorter sides given the hypotenuse and the other shorter side.

What is the significance of this result? Why did it hurt the brains of the *Ancient Greeks*?

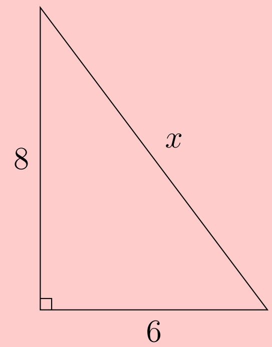
Solution: Because the answer is irrational. And what does it really mean to have an irrational line length?? How could you ever accurately draw something like that?!

4 Exercises

1. A door frame has height 1.7m and width 1m. Will a square piece of board 2m by 2m fit through the doorway?

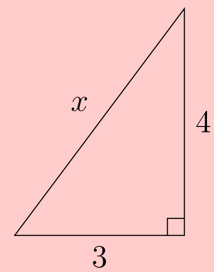
   Solution: The diagonal of the door frame is $\sqrt{1.7^2 + 1^2} = \sqrt{2.89 + 1} = \sqrt{3.89} \approx$ 1.97m, which is less than th side of the board (2m). Therefore, the board will not fit through the doorway.

2. 1



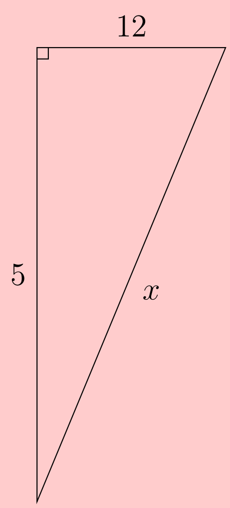
Solution: $x = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

4.



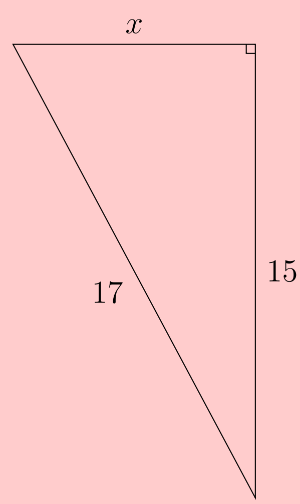
Solution: $x = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

3. 1



Solution: $x = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$

5.



Solution: $x = \sqrt{17^2 - 15^2} = \sqrt{289 - 225} = \sqrt{64} = 8$

The Converse

In the introduction we mentioned *the converse* of the **Pythagorean Theorem**. Let us recall that the theorem states: "in a *right-angled* triangle $a^2 + b^2 = c^2$, thus the converse becomes, *if* $a^2 + b^2 = c^2$, then the triangle must be *right-angled*.

Examples

Decide if the triangles with the following side lengths are **right-angled**.

1. 1 6,30,34 [1]

   Solution: $16^2 + 30^2 = 256 + 900 = 1156$ and $34^2 = 1156$. Since $16^2 + 30^2 = 34^2$, the triangle is right-angled.

2. 1 0,24,26 [1]

   Solution: $10^2 + 24^2 = 100 + 576 = 676$ and $26^2 = 676$. Since $10^2 + 24^2 = 26^2$, the triangle is right-angled.

3. 4 ,6,7 [1]

   Solution: $4^2 + 6^2 = 16 + 36 = 52$ and $7^2 = 49$. Since $4^2 + 6^2 \neq 7^2$, the triangle is not right-angled.

5 Introduction to Surds

What is a surd?

Solution: A surd is an irrational number that can be expressed in the form of a root, such as $\sqrt{2}$, which cannot be simplified to a rational number.

You must realise that a surd is an **exact value**, and anything that your calculator spits back at you is only an *approximation*. In your calculator $\sqrt{2} = 1.414213562$, but in truth $\sqrt{2}$ never terminates, it continues forever as it is *irrational*!
Let's round some surds to 3 decimal places:

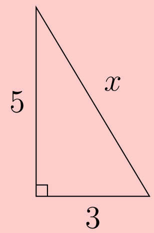
6 Examples

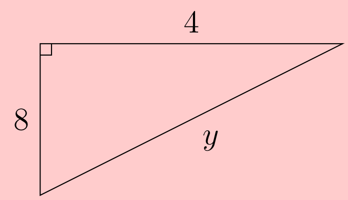
1. $\sqrt{19} =$ ____**4.359**____ 0.7in 2. $\sqrt{37} =$ ____**6.083**____ 0.7in 3. $\sqrt{161} =$ ____**12.688**____ 0.65in 4. $\sqrt{732} =$ ____**27.055**____ 0.65in [1]

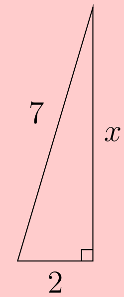
Now we shall find the *exact* lengths of the following sides. Realise that unlike before our side lengths are not whole or even *rational* numbers.

7 Exercises

1. 2. 1







Solution: $x = \sqrt{5.83^2 - 3^2} = \sqrt{34} \approx 5.831$

Solution: $y = \sqrt{8^2 - 4^2} = \sqrt{48} \approx 6.928$

3.

Solution: $x = \sqrt{7^2 - 2^2} = \sqrt{45} \approx 6.708$

4. A door frame has height 1.8m and width 1m. Will a square piece of board 2.1m wide fit through the opening? 2

Solution: The diagonal of the door frame is $\sqrt{1.8^2 + 1^2} = \sqrt{4.24} \approx 2.059$, which is greater than 2.1m, so the boar will fit.

5. A signwriter leans his ladder against a wall so that he can paint a sign. The wall is vertical and the ground in front of the wall is horizontal. The signwriter's ladder is 4m long. 3
If the signwriter wants the top of the ladder to be 3.8m above the grou when leaning against the wall, how far, correct to 1 decimal place should the foot of the ladder be placed from the wall?

Solution: The distance from the wall is $\sqrt{4^2 - 3.8^2} = \sqrt{0.36} \approx 0.6$m.

8 Simplifying Surds

For the moment we are finished with *geometric* thinking and are now going to begin playing with numbers and doing arithmetic on terms such as $\sqrt{121}$ and $\sqrt{48}$. You should be able to simplify the former $\sqrt{121} =$ ______**11**______, but the latter is a little more tricky.
Watch this: $\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$. This surd is now considered simplified.

9 Examples

Try to simplify the following:

1. $\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$
2. $\sqrt{108} = \sqrt{36 \times 3} = 6\sqrt{3}$
3. $\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$ 1
4. $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$ 1

10 Exercises

Now try going backwards:

1. $3\sqrt{7} = \sqrt{9 \times 7} = \sqrt{63}$
2. $5\sqrt{3} = \sqrt{25 \times 3} = \sqrt{75}$ 1

You will be doing much more practise with these new mathematical objects in the homework, so do not worry if it feels uncomfortable right now!

11 Addition & Subtraction of Surds

Let us now try to solve the problem from the start of the lesson: $\sqrt{8}+2\sqrt{2}$. We know from the previous section we can simplify $\sqrt{8}$ into ____ $2\sqrt{2}$ ____ and so our expression becomes __ $2\sqrt{2} + 2\sqrt{2}$ __. Now recall from last week that we can add **like terms** together such that our final answer becomes ____ $4\sqrt{2}$ ____.

The process for subtraction is exactly the same, and the way to get better at this arithmetic is by doing lots of exercises!

12 Exercises

12

1. (a) $$4\sqrt{7} + 5\sqrt{7} =$$

Solution: $9\sqrt{7}$

(b) $$6\sqrt{7} - 2\sqrt{7} =$$

Solution: $4\sqrt{7}$

(c) $$2\sqrt{2} + 5\sqrt{2} - 3\sqrt{2} =$$

Solution: $4\sqrt{2}$

(d) $$3\sqrt{5} + 2\sqrt{7} - \sqrt{5} + 4\sqrt{7} =$$

Solution: $2\sqrt{5} + 6\sqrt{7}$

(e) $$\sqrt{27} + 2\sqrt{5} + \sqrt{20} - 2\sqrt{3} =$$

Solution: $3\sqrt{3} + 2\sqrt{5} + 2\sqrt{5} - 2\sqrt{3} = \sqrt{3} + 4\sqrt{5}$

(f) $$\frac{\sqrt{3}}{2} + 3\sqrt{3} =$$

Solution: $\frac{1}{2}\sqrt{3} + \frac{6}{2}\sqrt{3} = \frac{7}{2}\sqrt{3}$

(g) $$6\sqrt{2} + 4\sqrt{2} =$$

Solution: $10\sqrt{2}$

(h) $$10\sqrt{7} - 5\sqrt{7} =$$

Solution: $5\sqrt{7}$

(i) $$3\sqrt{11} - 5\sqrt{11} =$$

Solution: $-2\sqrt{11}$

(j) $$2\sqrt{2} + 3\sqrt{2} + \sqrt{2} =$$

Solution: $6\sqrt{2}$

(k) $$10\sqrt{5} - \sqrt{5} - 6\sqrt{5} =$$

Solution: $3\sqrt{5}$

(l) $$\sqrt{7} - 8\sqrt{7} + 5\sqrt{7} =$$

Solution: $-2\sqrt{7}$

13 Multiplication & Division of Surds

Welcome to the last section for today: Multiplication and Division of Surds. It is exactly what you would expect:

$$4\sqrt{7} \times 2\sqrt{2} = 8\sqrt{14} \quad (1)$$

Realise that you do not need the terms to be *alike* to multiply and divide them.

14 Examples

1. $\sqrt{3} \times \sqrt{11} =$ 2

   Solution: $\sqrt{33}$

2. $\sqrt{15} \div \sqrt{3} =$

   Solution: $\sqrt{5}$

3. $3 \times 7\sqrt{5} =$ 2

   Solution: $21\sqrt{5}$

4. $\sqrt{2} \times 4 =$

   Solution: $4\sqrt{2}$

5. $\sqrt{6} \times \sqrt{6} =$

   Solution: 6

6. $(2\sqrt{6})^2 =$

   Solution: 24

7. $\sqrt{42} \div \sqrt{6} =$

   Solution: $\sqrt{7}$

8. $\sqrt{35} \div \sqrt{10} =$

   Solution: $\sqrt{3.5}$ or $\sqrt{\frac{7}{2}}$

15 Exercises

The distributive law is exactly the same as last week:

$$(a+b)(c+d) = ac + ad + bc + bd \quad (2)$$

Expand and simplify the following:

1. $2\sqrt{3}(4 + 3\sqrt{3}) =$ 1

Solution: $8\sqrt{3}+18$

2. $(3\sqrt{7}+1)(5\sqrt{7}-4) =$ 1

Solution: $15\sqrt{49}-12\sqrt{7}+5\sqrt{7}-4 = 105+5\sqrt{7}-12\sqrt{7}-4 = 101-7\sqrt{7}$

3. $(5\sqrt{2}-3)(2\sqrt{2}-4) =$ 1

Solution: $10\sqrt{4}-20\sqrt{2}-6\sqrt{2}+12 = 20-26\sqrt{2}+12 = 32-26\sqrt{2}$

4. $(3\sqrt{2}-4\sqrt{3})(5\sqrt{3}-\sqrt{2}) =$ 1

Solution: $15\sqrt{6}-3\sqrt{4}-20\sqrt{9}+4\sqrt{6} = 19\sqrt{6}-6-60 = -66+19\sqrt{6}$

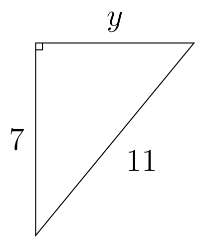
5. $(1-\sqrt{2})(3+2\sqrt{2}) =$ 1

Solution: $3+2\sqrt{4}-3\sqrt{2}-2\sqrt{4} = 3+4-3\sqrt{2}-4 = -3\sqrt{2}+3$

16 Homework

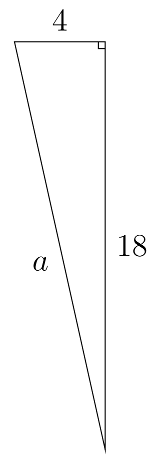
Section 2

1. (a)



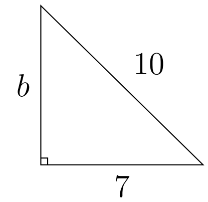
Solution: $y = \sqrt{11^2 - 7^2} = \sqrt{121 - 49} = \sqrt{72} = 8.49$

(b)



Solution: $a = \sqrt{18^2 - 4^2} = \sqrt{324 - 16} = \sqrt{308} = 17.55$

(c)



Solution: $b = \sqrt{10^2 - 7^2} = \sqrt{100 - 49} = \sqrt{51} = 7.14$

[3]

2. D etermine whether or not the following side-lengths form a right-angled triangle: [3]

(a) $4.5, 7, 7.5$

Solution: $7.5^2 \neq 4.5^2 + 7^2$, so it does not form a right-angled triangle.

(b) $6, 10, 12$

Solution: $12^2 = 6^2 + 10^2$, so it forms a right-angled triangle.

(c) $20, 21, 29$

Solution: $29^2 \neq 20^2 + 21^2$, so it does not form a right-angled triangle.

3. A s part of a design, an artist draws a circle passing through the four corners (vertices) of a square. [2]

(a) If the square has side lengths of 4cm, what is the radius, to the nearest millimetre, of the circle?

Solution: Radius $r = \frac{\sqrt{4^2+4^2}}{2} = \frac{\sqrt{32}}{2} = \frac{4\sqrt{2}}{2} = 2\sqrt{2} \approx 2.828$cm

(b) If the circle has a radius of 3cm, what are the side lengths, to the nearest millimetre, of the square?

Solution: Side length $s = \sqrt{2} \times r = \sqrt{2} \times 3 \approx 4.242$cm

4. A parent is asked to make some scarves for the local Scout troop. Two scarves can be made from one square piece of materia by cutting on the diagonal. If this diagonal side length is [2]

to be 100cm long, what must be the side length of the square piece of materia to the nearest mm?

Solution: Side length $s = \frac{100}{\sqrt{2}} \approx 70.71$cm

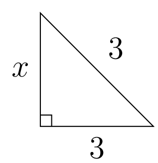
5. A girl planned to swim straight across a river of width 25m. After she had swum across the river, the girl found she had been swept 4m downstream. How far did she actually swim? Calculate your answer, in metres, correct to 1 decimal place. [2]

Solution: Distance swum $d = \sqrt{25^2 + 4^2} \approx 25.3$m

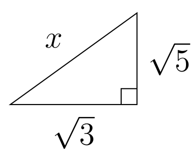
Section 3

1. U se Pythagoras' Theorem to find the value of x. Give your answer as a *surd* which has been simplified. [3]

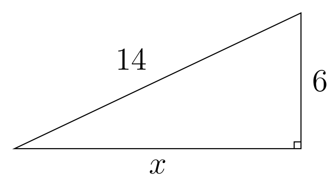
(a)



(b)



(c)



Solution: $x = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$

Solution: $x = \sqrt{\sqrt{5}^2 - \sqrt{3}^2} = \sqrt{5 - 3} = \sqrt{2}$

Solution: $x = \sqrt{14^2 - 6^2} = \sqrt{196 - 36} = \sqrt{160} = 4\sqrt{10}$

Section 4

1. S implify all of the following: [18]

(a) $(\sqrt{231})^2$

Solution: 231

(b) $(3\sqrt{5})^2$

Solution: $9 \times 5 = 45$

(c) $(2\sqrt{11})^2$

Solution: $4 \times 11 = 44$

(d) $\sqrt{3} \times \sqrt{5}$

Solution: $\sqrt{15}$

(e) $\sqrt{5} \times \sqrt{6}$

Solution: $\sqrt{30}$

(f) $\sqrt{6} \div \sqrt{2}$

Solution: $\sqrt{3}$

(g) $\frac{\sqrt{77}}{\sqrt{11}}$

Solution: $\sqrt{7}$

(h) $2 \times 3\sqrt{2}$

Solution: $6\sqrt{2}$

(i) $6 \times 5\sqrt{7}$

Solution: $30\sqrt{7}$

(j) $(\sqrt{2})^3$

Solution: $2\sqrt{2}$

(k) $(\sqrt{11})^2 + (\sqrt{2})^2$

Solution: $11 + 2 = 13$

(l) $(\sqrt{5})^2 + (\sqrt{11})^2$

Solution: $5 + 11 = 16$

(m) $\sqrt{45}$

Solution: $3\sqrt{5}$

(n) $\sqrt{54}$

Solution: $3\sqrt{6}$

(o) $\sqrt{126}$

Solution: $3\sqrt{14}$

(p) $\sqrt{72}$

Solution: $6\sqrt{2}$

(q) $\sqrt{96}$

Solution: $4\sqrt{6}$

(r) $\sqrt{200}$

Solution: $10\sqrt{2}$

2. E xpress the following surds as the square root of a whole number 4

(a) $2\sqrt{3}$

Solution: $\sqrt{12}$

(b) $2\sqrt{13}$

Solution: $\sqrt{52}$

(c) $4\sqrt{5}$

Solution: $\sqrt{80}$

(d) $12\sqrt{10}$

Solution: $\sqrt{1440}$

3. E valuate: 2

(a) $(\sqrt{\frac{2}{3}})^2$

Solution: $\frac{2}{3}$

(b) $\sqrt{\frac{16}{25}}$

Solution: $\frac{4}{5}$

Section 5

1. S implify: 16

(a) $6\sqrt{2} + 4\sqrt{2}$

Solution: $10\sqrt{2}$

(b) $10\sqrt{7} - 5\sqrt{7}$

Solution: $5\sqrt{7}$

(c) $3\sqrt{11} - 5\sqrt{11}$

Solution: $-2\sqrt{11}$

(d) $2\sqrt{2}+3\sqrt{2}+\sqrt{2}$

Solution: $6\sqrt{2}$

(e) $10\sqrt{5}-\sqrt{5}-6\sqrt{5}$

Solution: $3\sqrt{5}$

(f) $\sqrt{3}-2\sqrt{2}+2\sqrt{3}+\sqrt{2}$

Solution: $3\sqrt{3}-\sqrt{2}$

(g) $5\sqrt{14}+4\sqrt{6}+\sqrt{14}+3\sqrt{6}$

Solution: $6\sqrt{14}+7\sqrt{6}$

(h) $\sqrt{5}-3\sqrt{2}-4\sqrt{5}+7\sqrt{2}$

Solution: $-3\sqrt{5}+4\sqrt{2}$

(i) $\sqrt{50}+3\sqrt{2}$

Solution: $5\sqrt{2}+3\sqrt{2}=8\sqrt{2}$

(j) $\sqrt{48}+2\sqrt{3}$

Solution: $4\sqrt{3}+2\sqrt{3}=6\sqrt{3}$

(k) $\sqrt{8}+4\sqrt{2}+2\sqrt{18}$

Solution: $2\sqrt{2}+4\sqrt{2}+6\sqrt{2}=12\sqrt{2}$

(l) $\sqrt{8}+\sqrt{2}+\sqrt{18}$

Solution: $2\sqrt{2}+\sqrt{2}+3\sqrt{2}=6\sqrt{2}$

(m) $4\sqrt{5}-4\sqrt{20}-\sqrt{45}$

Solution: $4\sqrt{5}-8\sqrt{5}-3\sqrt{5}=-7\sqrt{5}$

(n)

$$\frac{\sqrt{7}}{2}-\frac{\sqrt{7}}{5}$$

Solution: $\frac{5\sqrt{7}-2\sqrt{7}}{10}=\frac{3\sqrt{7}}{10}$

(o)

$$\frac{3\sqrt{11}}{7}+\frac{2\sqrt{11}}{21}$$

Solution: $\frac{9\sqrt{11}+2\sqrt{11}}{21}=\frac{11\sqrt{11}}{21}$

(p) $\sqrt{80}-\sqrt{45}=\sqrt{x}$

Solution: $4\sqrt{5}-3\sqrt{5}=\sqrt{5}=\sqrt{x}$, so $x=5$

Section 6

1. S implify: [8]

(a) $4\sqrt{7}\times\sqrt{2}$

Solution: $4\sqrt{7}\times\sqrt{2}=4\sqrt{14}$

(b) $6\sqrt{3}\times 4\sqrt{7}$

Solution: $6\sqrt{3}\times 4\sqrt{7}=24\sqrt{21}$

(c) $12\sqrt{33}\div 3\sqrt{3}$

Solution: $12\sqrt{33}\div 3\sqrt{3}=4\sqrt{11}$

(d) $36\sqrt{15}\div 4\sqrt{3}$

Solution: $36\sqrt{15}\div 4\sqrt{3}=9\sqrt{5}$

(e) $7\sqrt{28}\div 4\sqrt{7}$

Solution: $7\sqrt{28} \div 4\sqrt{7} = 7\sqrt{4} = 14$

(f) $\sqrt{7} \times 2\sqrt{7}$

Solution: $\sqrt{7} \times 2\sqrt{7} = 2\sqrt{49} = 14$

(g) $7\sqrt{10} \times 3\sqrt{2}$

Solution: $7\sqrt{10} \times 3\sqrt{2} = 21\sqrt{20} = 42\sqrt{5}$

(h) $\sqrt{2}(2\sqrt{2} - \sqrt{6})$

Solution: $\sqrt{2}(2\sqrt{2} - \sqrt{6}) = 2 \cdot 2 - \sqrt{12} = 4 - 2\sqrt{3}$

2. E xpand and simplify: 12

(a) $5\sqrt{5}(4\sqrt{2} - 3)$

Solution: $5\sqrt{5}(4\sqrt{2} - 3) = 20\sqrt{10} - 15\sqrt{5}$

(b) $2\sqrt{3}(3\sqrt{3} - 5)$

Solution: $2\sqrt{3}(3\sqrt{3} - 5) = 6 \cdot 3 - 10\sqrt{3} = 18 - 10\sqrt{3}$

(c) $3\sqrt{7}(2 - \sqrt{14})$

Solution: $3\sqrt{7}(2-\sqrt{14}) = 6\sqrt{7}-3\cdot 7 = 6\sqrt{7} - 21$

(d) $\sqrt{2}(2\sqrt{2} - \sqrt{6})$

Solution: $\sqrt{2}(2\sqrt{2} - \sqrt{6}) = 4 - 2\sqrt{3}$

(e) $(4\sqrt{5} + 1)(3\sqrt{5} + 2)$

Solution: $(4\sqrt{5}+1)(3\sqrt{5}+2) = 12\cdot 5 + 8\sqrt{5} + 3\sqrt{5} + 2 = 60 + 11\sqrt{5}$

(f) $(3\sqrt{2} + 2)(3\sqrt{2} - 1)$

Solution: $(3\sqrt{2}+2)(3\sqrt{2}-1) = 9\cdot 2 + 6\sqrt{2} - 3\sqrt{2} - 2 = 18 + 3\sqrt{2}$

(g) $(2\sqrt{3} - 4)(3\sqrt{3} + 5)$

Solution: $(2\sqrt{3}-4)(3\sqrt{3}+5) = 6\cdot 3 + 10\sqrt{3} - 12\sqrt{3} - 20 = 18 - 2\sqrt{3} - 20 = -2 - 2\sqrt{3}$

(h) $(7\sqrt{2} + 5)^2$

Solution: $(7\sqrt{2}+5)^2 = 49\cdot 2 + 70\sqrt{2} + 25 = 98 + 70\sqrt{2} + 25 = 123 + 70\sqrt{2}$

(i) $(3\sqrt{5} + 2)(\sqrt{2} + 3)$

Solution: $(3\sqrt{5}+2)(\sqrt{2}+3) = 3\sqrt{10} + 9\sqrt{5} + 2\sqrt{2} + 6$

(j) $(4 + 2\sqrt{3})(2\sqrt{7} - 5)$

Solution: $(4+2\sqrt{3})(2\sqrt{7}-5) = 8\sqrt{7} - 20 + 4\sqrt{21} - 10\sqrt{3}$

(k) $(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5})$

Solution: $(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5}) = 7 - 5 = 2$

(l) $(4\sqrt{7} - \sqrt{5})(2\sqrt{5} + \sqrt{7})$

Solution: $(4\sqrt{7} - \sqrt{5})(2\sqrt{5} + \sqrt{7}) = 8\sqrt{35} + 4\cdot 7 - 2\cdot 5 - \sqrt{35} = 28 - 10 + 7\sqrt{35}$

3. **Challenge:**

4. I f $x = \sqrt{2} - 1$ and $y = \sqrt{3} + 1$, find: 2

Solution: $x + y = (\sqrt{2} - 1) + (\sqrt{3} + 1) = \sqrt{2} + \sqrt{3}$

5. [2]

$$\frac{6\sqrt{10}}{x+1}$$

Solution:

$$\frac{6\sqrt{10}}{\sqrt{2}} = 6\sqrt{5}$$

6. [2]

$$x + \frac{1}{x}$$

Solution:

$$\sqrt{2} - 1 + \frac{1}{\sqrt{2} - 1} = \sqrt{2} - 1 + \frac{\sqrt{2} + 1}{1} = 2\sqrt{2}$$

Marker's use only.

SECTION	1	2	3	4	5	6	HW	**Total**
MARKS	$\overline{0}$	$\overline{10}$	$\overline{12}$	$\overline{6}$	$\overline{12}$	$\overline{13}$	$\overline{81}$	$\overline{134}$