Year 9 Mathematics
Topic 4
Factorisation

PEN Education

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Contents

1 Introduction

Today we will cover the theory of **factorisation**. This is the process of taking something like $x^2 + 5x + 6$ and transforming it into $(x + 2)(x + 3)$. This looks familiar because it is the reverse process to *expansion*, which is what we learned in our **Topic 1** lesson.

1. Why do you think it is important to learn the factorisation technique? [1]

> **Solution:** So that we are able to break something down into its components and then maybe if we are lucky, cancel these common components out to make the expression more simple.

2. What is a silly, real life example of factorisation? [1]

Solution: Considering anything to be broken down into its components. An outfit for example.

Here is a box with a number of **buzz words** from today's class. As you come to understand these words, cross them off here.

common factor difference of two squares monic non-monic quadratic perfect square factorisation expansion

2 Common Factors

We can factorise expressions entirely by realising that there is a some term which can be pulled out of all the terms. Consider $4a + 12$. Here we have _____**3**_____ common, and so this can be factorised as __$4(a+3)$__. Similarly, see for the following examples what term can be extracted from all the other ones.

Examples:

1. Factorise: [8]

(a) $12x^2 + 3x$

Solution: $12x^2 + 3x = 3\left(4x^2 + x\right)$

(b) $36ab - 27a$

Solution: $36ab - 27a = 9(4ab - 3a) = 3x(4x + 1) = 9a(4b - 3)$

(c) $3x + 9$

Solution: $3x + 9 = 3(x + 3)$

(d) $-7a^2 - 49$

Solution: $-7a^2 - 49 = -7 \times a^2 + (-7) \times 7$

(e) $7a^2 + 63ab$

Solution: $7a^2 + 63ab = 7a(a + 9b) = -7\left(a^2 + 7\right)\ 7\left(-a^2 - 7\right)$ is also correct.

(f) $5pq^2 + 10p^2q + 25p^2q^2$

Solution: $5pq^2 + 10p^2q + 25p^2q^2 = 5pq(q + 2p + 5pq)$

2.1 Exercises:

1. Complete each factorisation. [8]

(a) $12x = 12 \times$ **x**

(b) $24a = 12 \times$ **2a**

(c) $15ac = 5c \times$ **3a**

(d) $y^2 = y \times$ **y**

(e) $24a^2 = 6a \times$ **4a**

(f) $-6b^2 = 2b \times$ **-3b**

(g) $4x^2y = 2x \times$ **2xy**

(h) $12m^2n = 3mn \times$ **4m**

2. Fill in the blanks by finding the missing factors. [4]

(a) $12a + 18 = 6 \times$ **(2a+3)**

(b) $20mn - 15n = 5n \times$ **(4m-3)**

(c) $a^2 + 4a = (a + 4) \times$ **a**

(d) $6yz^2 - 18yz = 3yz \times$ **(2z-6)**

3. Factorise: [6]

(a) $6x + 24 =$ **6(x+4)**

(b) $5a + 15 =$ **5(a+3)**

(c) $y^2 + xy =$ **y(y+x)**

(d) $4x + 24 =$ **4(x+6)**

(e) $y^2 - 3y =$ **y(y-3)**

(f) $-14a - 21 =$ **-7(2a+3)**

4. Factorise: [8]

(a) $4ab + 16a =$ **4a(b+4)**

(b) $12a^2 + 8a =$ **4a(3a+2)**

(c) $15a^2b^2 + 10ab^2 = 5ab^2(3a + 2)$

(d) $4a^2 + 6a =$ **2a(2a+3)**

(e) $3b - 6b^2 =$ **-3b(2b-1)**

(f) $4x^2 - 6xy =$ **2x(2x-3y)**

(g) $18y - 9y^2 =$ **9y(2-y)**

(h) $4a - 6ab^2 = 2a(2 - 3b^2)$

(i) $14mn^2 - 21m^2n =$ **7mn(2n-3m)**

(j) $6pq^2 - 21qp^2 =$ **3pq(2q-7p)**

(k) $-10b^2 + 5b =$ **5b(-2b+1)**

(l) $-4pq + 16p^2 =$ **4p(-q+4p)**

(m) $-16a^2b - 8ab =$ **-8ab(2a+1)**

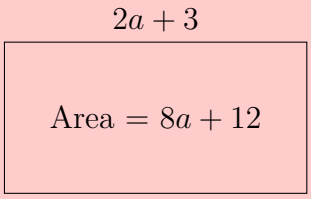
(n) $-8a^2b^2 - 2ab =$ **-2ab(4ab+1)**

(o) $-5x^2y + 30x = 5x(-x^2y + 6)$

(p) $12xy^2 - 3x^2y =$ **3xy(4y-x)**

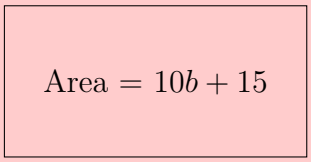
5. In each part, an expression for the area of the rectangle has been given. Find an expression for the missing side length. [3]

(a)



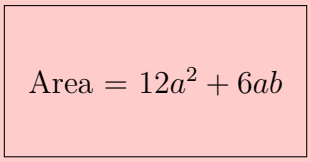
Solution: 4

(b)



Solution: $2b + 3$

(c)



Solution: $2a + b$

6. Factorise: [4]

(a) $4a^2b - 2ab + 8ab^2$

Solution: $2ab(2a - 1 + 4b)$

(b) $7ab + 14a^2 + 21b$

Solution: $7(2a^2 + ab + 3b)$

(c) $5a^2b + 3ab + 4ab^2$

Solution: $ab(5a + 3 + 4b)$

(d) $5p^2q^2 + 10pq^2 + 15p^2q$

Solution: $5pq(pq + 2q + 3p)$

3 Difference of Two Squares

Continueing with the theme of using our algebraic identities *backwards*, we have

$$(a + b)(a - b) = a^2 - b^2,$$

Which becomes:

$$a^2 - b^2 = (a + b)(a - b)$$

Now we can write expressions such as $x^2 - 9$ in the factored form of $\underline{(x + 3)(x - 3)}$.

3.1 Examples:

1. Factorise: [14]

(a) $25 - y^2$

Solution: $25 - y^2 = 5^2 - y^2$

$= (5 + y)(5 - y)$

(b) $4x^2 - 9$

Solution: $4x^2 - 9 = (2x)^2 - 3^2$

$= (2x + 3)(2x - 3)$

(c) $3a^2 - 27$

Solution: $3a^2 - 27 = 3\,(a^2 - 9)$

(d) $-16 + 9x^2$

Solution: $-16 + 9x^2 = 9x^2 - 16 = 3\,(a^2 - 3^2) = (3x)^2 - 4^2 = 3(a+3)(a-3) = (3x - 4)(3x + 4)$

(e) $x^2 - 16$

Solution: $x^2 - 16 = (x + 4)(x - 4)$

(f) $(2x)^2 - 25$

Solution: $(2x)^2 - 25 = (2x+5)(2x-5)$

(g) $9x^2 - 4$

Solution: $9x^2 - 4 = (3x + 2)(3x - 2)$

(h) $1 - 4a^2$

Solution: $1 - 4a^2 = (1 + 2a)(1 - 2a)$

(i) $3x^2 - 48$

Solution: $3x^2 - 48 = 3(x^2 - 16) = 3(x + 4)(x - 4)$

(j) $10x^2 - 1000$

Solution: $10x^2 - 1000 = 10(x^2 - 100) = 10(x + 10)(x - 10)$

(k) $3 - 12b^2$

Solution: $3 - 12b^2 = 3(1 - 4b^2) = 3(1 + 2b)(1 - 2b)$

(l) $45m^2 - 125n^2$

Solution: $45m^2 - 125n^2 = 5(9m^2 - 25n^2) = 5(3m + 5n)(3m - 5n)$

(m) $-25 + x^2$

Solution: $-25 + x^2 = x^2 - 25 = (x + 5)(x - 5)$

(n) $-81x^2 + 16$

Solution: $-81x^2 + 16 = 16 - 81x^2 = (4 + 9x)(4 - 9x)$

2. Use the factorisation of the difference of two squares to evaluate the following. One has been done for you. [5]

$$17^2 - 3^2 = (17 + 3)(17 - 3)$$
$$= 20 \times 14$$
$$= 280$$

(a) $23^2 - 7^2$

Solution: $23^2 - 7^2 = (23+7)(23-7) = 30 \times 16 = 480$

(c) $11.3^2 - 8.7^2$

Solution: $11.3^2 - 8.7^2 = (11.3 + 8.7)(11.3 - 8.7) = 20 \times 2.6 = 52$

(b) $94^2 - 6^2$

Solution: $94^2 - 6^2 = (94+6)(94-6) = 100 \times 88 = 8800$

(d) $4^2 - 3^2$

Solution: $4^2 - 3^2 = (4 + 3)(4 - 3) = 7 \times 1 = 7$

Note: You can always check your factorisation is correct by expanding your result and checking to see if that is the expression you started with!

3.2 Exercises:

1. Factorise: [16]

(a) $x^2 - 49$

Solution: $x^2 - 49 = (x + 7)(x - 7)$

(b) $(3x)^2 - 16$

Solution: $(3x)^2 - 16 = (3x + 4)(3x - 4)$

(c) $16y^2 - 49$

Solution: $16y^2 - 49 = (4y + 7)(4y - 7)$

(d) $9 - 16y^2$

Solution: $9 - 16y^2 = (3 + 4y)(3 - 4y)$

(e) $a^2 - 121$

Solution: $a^2 - 121 = (a + 11)(a - 11)$

(f) $(4x)^2 - 1$

Solution: $(4x)^2 - 1 = (4x + 1)(4x - 1)$

(g) $100a^2 - 49b^2$

Solution: $100a^2 - 49b^2 = (10a + 7b)(10a - 7b)$

(h) $25a^2 - 100b^2$

Solution: $25a^2 - 100b^2 = (5a + 10b)(5a - 10b)$

(i) $d^2 - 400$

Solution: $d^2 - 400 = (d + 20)(d - 20)$

(j) $4x^2 - 100$

Solution: $4x^2 - 100 = (2x + 10)(2x - 10)$

(k) $5x^2 - 45$

Solution: $5x^2 - 45 = 5(x^2 - 9) = 5(x + 3)(x - 3)$

(l) $7x^2 - 63$

Solution: $7x^2 - 63 = 7(x^2 - 9) = 7(x + 3)(x - 3)$

(m) $8x^2 - 50$

Solution: $8x^2 - 50 = 2(4x^2 - 25) = 2(2x + 5)(2x - 5)$

(m) $20 - 5y^2$

Solution: $20 - 5y^2 = 5(4 - y^2) = 5(2 + y)(2 - y)$

(t) $23^2 - 3^2$

Solution: $23^2 - 3^2 = (23+3)(23-3) = 26 \times 20 = 520$

(n) $27a^2 - 12b^2$

Solution: $27a^2 - 12b^2 = 3(9a^2 - 4b^2) = 3(3a + 2b)(3a - 2b)$

(u) $1.8^2 - 0.2^2$

Solution: $1.8^2 - 0.2^2 = (1.8 + 0.2)(1.8 - 0.2) = 2 \times 1.6 = 3.2$

(o) $27a^2 - 192l^2$

Solution: $27a^2 - 192l^2 = 3(9a^2 - 64l^2) = 3(3a + 8l)(3a - 8l)$

(v) $92.6^2 - 7.4^2$

Solution: $92.6^2 - 7.4^2 = (92.6 + 7.4)(92.6 - 7.4) = 100 \times 85.2 = 8520$

(p) $-8x^2 + 32y^2$

Solution: $-8x^2 + 32y^2 = 32y^2 - 8x^2 = 8(4y^2 - x^2) = 8(2y + x)(2y - x)$

(w) $5^2 - 4^2$

Solution: $5^2 - 4^2 = (5 + 4)(5 - 4) = 9 \times 1 = 9$

(q) $-4 + 9x^2$

Solution: $-4 + 9x^2 = 9x^2 - 4 = (3x + 2)(3x - 2)$

(x) $9^2 - 8^2$

Solution: $9^2 - 8^2 = (9 + 8)(9 - 8) = 17 \times 1 = 17$

(r) $-100x^2 + 9$

Solution: $-100x^2 + 9 = 9 - 100x^2 = (3 + 10x)(3 - 10x)$

(y) $6^2 - 5^2$

Solution: $6^2 - 5^2 = (6 + 5)(6 - 5) = 11 \times 1 = 11$

(s) $-36x^2 + 400$

Solution: $-36x^2 + 400 = 400 - 36x^2 = (20 + 6x)(20 - 6x)$

(z) $10^2 - 9^2$

Solution: $10^2 - 9^2 = (10+9)(10-9) = 19 \times 1 = 19$

4 Simple Quadratics

The nomenclature "simple" quadratic means that the coefficient of x^2 is just 1. Let us recall some definitions:

1. **Definition 1** [1]

Nomenclature:

Solution: the terminology for a specific topic

2. **Definition 2** 1

Quadratic:

Solution: a polynomial whose highest power is 2

3. **Definition 3** 1

Coefficient:

Solution: the number before the variable

The Technique:
Let us take a concrete example: $x^2 - 3x - 18$. To factorise this monic quadratic we need to find two numbers that multiply to give -18 and add to give -3. Such numbers will be -6 and 3. Thus we can write $x^2 - 3x - 18 = (x + 3)(x - 6)$

Examples:

1. Factorise: 2

   (a) $x^2 + 8x + 16$

   Solution: $x^2 + 8x + 16 = (x + 4)(x + 4)$

   $= (x + 4)^2$

   (b) $x^2 + 5x + 6$

   Solution: $x^2 + 5x + 6 = (x + 2)(x + 3)$

   (c) $x^2 + 7x + 10$

   Solution: $x^2 + 7x + 10 = (x + 2)(x + 5)$

   (d) $x^2 + 9x + 14$

   Solution: $x^2 + 9x + 14 = (x + 2)(x + 7)$

Exercises:

1. Factorise: 4

(a) $x^2 + 9x + 20$

Solution: $x^2+9x+20 = (x+4)(x+5)$

(b) $x^2 + 12x + 32$

Solution: $x^2+12x+32 = (x+4)(x+8)$

(c) $x^2 + 20x + 75$

Solution: $x^2+20x+75 = (x+5)(x+15)$

(d) $x^2 + 15x + 56$

Solution: $x^2+15x+56 = (x+7)(x+8)$

(e) $x^2 - 5x + 6$

Solution: $x^2-5x+6 = (x-2)(x-3)$

(f) $x^2 - 17x + 30$

Solution: $x^2-17x+30 = (x-2)(x-15)$

(g) $x^2 - 9x + 14$

Solution: $x^2-9x+14 = (x-2)(x-7)$

(h) $x^2 - 15x + 44$

Solution: $x^2-15x+44 = (x-4)(x-11)$

(i) $x^2 - 18x + 80$

Solution: $x^2-18x+80 = (x-10)(x-8)$

(j) $x^2 - 14x + 40$

Solution: $x^2-14x+40 = (x-4)(x-10)$

(k) $x^2 - 30x + 56$

Solution: $x^2-30x+56 = (x-8)(x-7)$

(l) $x^2 + x - 6$

Solution: $x^2+x-6 = (x-2)(x+3)$

(m) $x^2 + x - 30$

Solution: $x^2+x-30 = (x-5)(x+6)$

(n) $x^2 - 5x - 14$

Solution: $x^2-5x-14 = (x+2)(x-7)$

(o) $x^2 - 7x - 44$

Solution: $x^2-7x-44 = (x+4)(x-11)$

(p) $x^2 + 2x - 80$

Solution: $x^2+2x-80 = (x-8)(x+10)$

(q) $x^2 + 3x - 40$

Solution: $x^2+3x-40 = (x-5)(x+8)$

(r) $x^2 + 4x - 21$

Solution: $x^2+4x-21 = (x-3)(x+7)$

(m) $x^2 - x - 56$

Solution: $(x-8)(x+7)$

(n) $x^2 - 3x + 2$

Solution: $x^2-3x+2 = (x-1)(x-2)$

(o) $x^2 - 3x - 10$

Solution: $x^2-3x-10 = (x+2)(x-5)$

(p) $x^2 - 5x - 14$

Solution: $x^2-5x-14 = (x+2)(x-7)$

(q) $x^2 - 5x + 4$

Solution: $x^2-5x+4 = (x-1)(x-4)$

(r) $x^2 - x - 12$

Solution: $x^2-x-12 = (x+3)(x-4)$

(s) $x^2 + 3x - 10$

Solution: $x^2+3x-10 = (x-2)(x+5)$

(t) $x^2 + 6x + 9$

Solution: $x^2+6x+9 = (x+3)(x+3)$

$= (x+3)^2$

5 Perfect Squares

The expansion of a perfect square has a special form. For example:

$$\begin{aligned}(x+3)^2 &= (x+3)(x+3)\\ &= x^2+6x+9\\ &= x^2+2\times(3x)+3^2\end{aligned}$$

5.1 Theorem

$$a^2+2ab+b^2 = (a+b)^2 \quad (1)$$
$$a^2-2ab+b^2 = (a-b)^2 \quad (2)$$

Examples:

1. (a) $x^2 + 8x + 16$

Solution: $x^2 + 8x + 16 = x^2 + 2\times 4x + 4^2$

$= (x+4)^2$

(b) $x^2 - 10x + 25$

3

Solution: $x^2 - 10x + 25 = x^2 - 2 \times 5x + 5^2$

$$= (x-5)^2$$

(c) $x^2 + 11x + \frac{121}{4}$

Solution: $x^2 + 11x + \frac{121}{4} = x^2 + 2 \times \frac{11}{2}x + \left(\frac{11}{2}\right)^2$

$$= \left(x + \frac{11}{2}\right)^2$$

2. Identify the simple quadratic expression that cannot be factorised as a perfect square. 2

(a) $x^2 + 4x + 4$

Solution: Can be factorised as $(x+2)^2$.

(b) $x^2 - 6x + 12$

Solution: Cannot be factorised as a perfect square because $b^2 - 4ac = 36 - 48 < 0$.

(c) $x^2 - 12x + 36$

Solution: Can be factorised as $(x-6)^2$.

(d) $x^2 - 10x + 25$

Solution: Can be factorised as $(x-5)^2$.

(e) $x^2 + 6x + 9$

Solution: Can be factorised as $(x+3)^2$.

(f) $x^2 + 5x + \frac{25}{4}$

Solution: Can be factorised as $\left(x + \frac{5}{2}\right)^2$.

(g) $x^2 - 8x - 16$

Solution: Cannot be factorised as a perfect square because $b^2 - 4ac = 64+64 > 0$ and is not a perfect square.

(h) $x^2 - 14x + 49$

Solution: Can be factorised as $(x-7)^2$.

Exercises:

1. (a) $x^2 + 12x + 36$ [8]

Solution: $x^2 + 12x + 36 = (x + 6)^2$

(b) $a^2 - 4a + 4$

Solution: $a^2 - 4a + 4 = (a - 2)^2$

(c) $x^2 - 9x + \frac{81}{4}$

Solution: $x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)^2$

(d) $x^2 + 8x + 16 = (x + \ldots)^{\cdots}$

Solution: $x^2 + 8x + 16 = (x + 4)^2$

(e) $x^2 - \ldots + \ldots = (x - 9)^2$

Solution: $x^2 - 18x + 81 = (x - 9)^2$

(f) $x^2 - 8x + 16$

Solution: $x^2 - 8x + 16 = (x - 4)^2$

(g) $m^2 - 26m + 169$

Solution: $m^2 - 26m + 169 = (m - 13)^2$

(h) $x^2 + 13x + \frac{169}{4}$

Solution: $x^2 + 13x + \frac{169}{4} = \left(x + \frac{13}{2}\right)^2$

2. A brick company provides rectangular and square pavers. [4]

(a) Draw a diagram to show how two different square pavers of side lengths a and b respectively, and two identical rectangular pavers with dimensions $a \times b$, can be arranged into a square.

Solution:

(b) How many of each type of paver enables you to pave a square area of side length $a + 3b$? Draw a diagram to illustrate how this can be done.

Solution: You would need 1 square paver of side length a, 9 square pavers of side length b, and 6 rectangular pavers of dimensions $a \times b$.

6 Quadratics with Common Factors

The quadratics for this section are a little less tame. The x^2 term now possesses a coefficient which we will be trying to pull out of the entire expression to use our techniques from section 4 on the new expression within the parenthesis.

6.1 Examples:

1. Factorise: [3]

(a) $3x^2 + 9x + 6$

Solution: $3x^2 + 9x + 6 = 3\left(x^2 + 3x + 2\right)$ (Take out the common factor.)

$$= 3(x + 2)(x + 1)$$

(b) $6x^2 - 54$

Solution: $6x^2 - 54 = 6\left(x^2 - 9\right)$

$$= 6(x + 3)(x - 3)$$

(c) $-x^2 - x + 2$

Solution: $-x^2 - x + 2 = -\left(x^2 + x - 2\right)$
(Factor -1 from each term.)

$$= -(x + 2)(x - 1)$$

10

2. (a) $2x^2 + 14x + 24$

Solution: $2x^2 + 14x + 24 = 2(x^2 + 7x + 12)$

$= 2(x + 3)(x + 4)$

(b) $4x^2 - 24x + 36$

Solution: $4x^2 - 24x + 36 = 4(x^2 - 6x + 9)$

$= 4(x - 3)^2$

(c) $4x^2 - 4x + 48$

Solution: $4x^2 - 4x + 48 = 4(x^2 - x + 12)$

(This quadratic does not factor over the integers.)

(d) $3x^2 + 9x - 120$

Solution: $3x^2 + 9x - 120 = 3(x^2 + 3x - 40)$

$= 3(x + 8)(x - 5)$

(e) $2x^2 - 4x - 96$

Solution: $2x^2 - 4x - 96 = 2(x^2 - 2x - 48)$

$= 2(x - 8)(x + 6)$

(f) $3x^2 - 18x + 27$

Solution: $3x^2 - 18x + 27 = 3(x^2 - 6x + 9)$

$= 3(x - 3)^2$

(g) $4x^2 - 16$

Solution: $4x^2 - 16 = 4(x^2 - 4)$

$= 4(x + 2)(x - 2)$

(h) $3a^2 - 27$

Solution: $3a^2 - 27 = 3(a^2 - 9)$

$= 3(a + 3)(a - 3)$

(i) $27x^2 - 3y^2$

Solution: $27x^2 - 3y^2 = 3(9x^2 - y^2)$

$= 3(3x + y)(3x - y)$

(j) $128 - 2x^2$

Solution: $128 - 2x^2 = -2(x^2 - 64)$

$= -2(x + 8)(x - 8)$

(k) $\frac{1}{4}a^2 - 9$

Solution: $\frac{1}{4}a^2 - 9 = \left(\frac{1}{2}a\right)^2 - 3^2$

$$= \left(\frac{1}{2}a + 3\right)\left(\frac{1}{2}a - 3\right)$$

(l) $-x^2 - 8x - 12$

Solution: $-x^2 - 8x - 12 = -1(x^2 + 8x + 12)$

$$= -(x + 6)(x + 2)$$

(m) $9 + 8x - x^2$

Solution: $9+8x-x^2 = -(x^2-8x-9)$

$$= -(x - 9)(x + 1)$$

(n) $-x^2 + 3x + 40$

Solution: $-x^2 + 3x + 40 = -1(x^2 - 3x - 40)$

$$= -(x - 8)(x + 5)$$

(o) $11x - x^2 - 24$

Solution: $11x - x^2 - 24 = -(x^2 - 11x + 24)$

$$= -(x - 8)(x - 3)$$

(p) $-16x - 63 - x^2$

Solution: $-16x - 63 - x^2 = -(x^2 + 16x + 63)$

$$= -(x + 7)(x + 9)$$

6.2 Exercises:

1. Factorise: 16

(a) $3x^2 + 24x + 36$

Solution: $3x^2 + 24x + 36 = 3(x^2 + 8x + 12)$

$= 3(x + 6)(x + 2)$

(b) $7x^2 + 14x + 7$

Solution: $7x^2+14x+7 = 7(x^2+2x+1)$

$= 7(x + 1)^2$

(c) $2x^2 - 18x + 36$

Solution: $2x^2 - 18x + 36 = 2(x^2 - 9x + 18)$

$= 2(x - 6)(x - 3)$

(d) $3x^2 - 3x - 90$

Solution: $3x^2 - 3x - 90 = 3(x^2 - x - 30)$

$= 3(x - 6)(x + 5)$

(e) $5x^2 + 65x + 180$

Solution: $5x^2 + 65x + 180 = 5(x^2 + 13x + 36)$

$= 5(x + 9)(x + 4)$

(f) $5x^2 - 20x + 20$

Solution: $5x^2 - 20x + 20 = 5(x^2 - 4x + 4)$

$= 5(x - 2)^2$

(g) $2x^2 - 18$

Solution: $2x^2 - 18 = 2(x^2 - 9)$

$= 2(x + 3)(x - 3)$

(h) $6x^2 - 600$

Solution: $6x^2 - 600 = 6(x^2 - 100)$

$= 6(x + 10)(x - 10)$

(i) $45 - 5b^2$

Solution: $45 - 5b^2 = -5(b^2 - 9)$

$= -5(b + 3)(b - 3)$

(j) $\frac{1}{2}a^2 - 2b^2$

Solution: $\frac{1}{2}a^2 - 2b^2 = \frac{1}{2}(a^2 - 4b^2)$

$= \frac{1}{2}(a + 2b)(a - 2b)$

(k) $\frac{1}{5}x^2 - 20$

Solution: $\frac{1}{5}x^2 - 20 = \frac{1}{5}(x^2 - 100)$

$= \frac{1}{5}(x + 10)(x - 10)$

(l) $12 - 11x - x^2$

Solution: $12 - 11x - x^2 = -(x^2 + 11x - 12)$

$= -(x - 1)(x + 12)$

(a) $-x^2 - 4x - 4$

Solution: $-x^2 - 4x - 4 = -1(x^2 + 4x + 4)$

$= -(x + 2)^2$

(b) $42 + x - x^2$

Solution: $42+x-x^2 = -(x^2-x-42)$

$= -(x - 7)(x + 6)$

(c) $-3x^2 - 30x + 72$

Solution: $-3x^2-30x+72 = -3(x^2+10x - 24)$

$= -3(x - 2)(x + 12)$

(d) $-x^2 - 35 + 12x$

Solution: $-x^2-35+12x = -1(x^2-12x + 35)$

$= -(x - 7)(x - 5)$

7 Homework

7.1 Common Factors

1. Fill in the gaps: [4]

   (a) $36ab = 9a \times$ **4b**

   (b) $6y^2 = 3y \times$ **2y**

   (c) $8a^2b = 2ab \times$ **4a**

   (d) $25a^2b^2 = 5ab \times$ **5ab**

2. Factorise: [2]

   (a) $16ab + 10b^2 - 2a^2b$

   Solution: $2b(8a + 5b - a^2)$

   (b) $2x^2 + 4x$

   Solution: $2x(x + 2)$

3. Fill in the blanks: [5]

   (a) $15p - 10 = 5 \times$ **3p-2**

   (b) $20mn - 15n = 5n \times$ **4m-3**

   (c) $b^2 - 10b = b \times$ **b-10**

   (d) $6yz^2 - 18yz = yz \times$ **6z-18**

   (e) $6yz^2 - 18yz = 6yz \times$ **z-3**

4. Pull out the common factor: [6]

   (a) $ac + 5c$

   Solution: $c(a + 5)$

   (b) $a^2 + a$

   Solution: $a(a + 1)$

   (c) $7a - 63$

   Solution: $7(a - 9)$

   (d) $9a + 36$

   Solution: $9(a + 4)$

   (e) $-6y - 9$

   Solution: $-3(2y + 3)$

   (f) $-4 - 12b$

   Solution: $-4(1 + 3b)$

5. Factorise: [8]

   (a) $18m^2n + 9mn^2$

   Solution: $9mn(2m + n)$

   (b) $8a^2 + 12ab$

   Solution: $4a(2a+3b)$

   (c) $9mn - 12m^2n$

   Solution: $-3mn(4m - 3)$

   (d) $6xy - 4x^2$

   Solution: $2x(3y - 2x)$

   (e) $10ab^2 - 25a^2b$

   Solution: $-5ab(5a - 2b)$

   (f) $-x^2y - 3xy$

   Solution: $-xy(x+3)$

   (g) $18p^2 - 4pq$

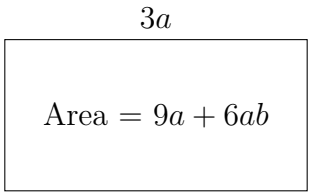
   Solution: $2p(9p-2q)$

   (h) $-25m^2n^2 - 10mn^2$

   Solution: $-5mn^2(5m + 2)$

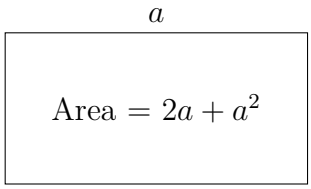
6. In each part, an expression for the area of the rectangle has been given. Find an expression for the missing side length. [3]

(a)



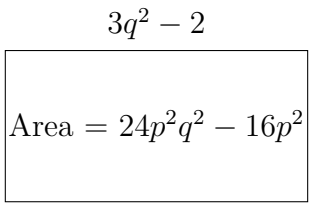
Solution: $3 + 2b$

(b)



Solution: $2 + a$

(c)



Solution: $8p^2$

7. Factorise: [4]

(a) $4m^2n - 4mn + 16n^2$

Solution: $4n(m^2 - m + 4n)$

(b) $2m^2 + 4mn + 6n$

Solution: $2(m^2 + 2mn + 3n)$

(c) $6a + 8ab + 10ab^2$

Solution: $2a(3 + 4b + 5b^2)$

(d) $5\ell^2 - 15\ell m - 20m^2$

Solution: $-5(\ell^2 - 3\ell m + 4m^2)$

7.2 Difference of Two Squares

1. Factorise: [16]

(a) $d^2 - 400$

Solution: $(d - 20)(d + 20)$

(b) $(5m)^2 - 9$

Solution: $(5m - 3)(5m + 3)$

(c) $64m^2 - 81p^2$

Solution: $(8m - 9p)(8m + 9p)$

(d) $-9 + x^2$

Solution: $(x - 3)(x + 3)$

(e) $6x^2 - 24$

Solution: $6(x^2 - 4)$ $6(x - 2)(x + 2)$

(f) $12m^2 - 75$

Solution: $3(4m^2 - 25)$ $3(2m - 5)(2m + 5)$

(g) $16x^2 - 100y^2$

Solution: $(4x - 10y)(4x + 10y)$

(h) $-200p^2 + 32q^2$

Solution: $-8(25p^2 - 4q^2)$ $-8(5p - 2q)(5p + 2q)$

(i) $-9x^2 + 4$

Solution: $-(3x-2)(3x+2)$

(j) $-18+50x^2$

Solution: $2(25x^2-9)$ $2(5x-3)(5x+3)$

(k) $-175+28x^2$

Solution: $-7(25-4x^2)$ $-7(5-2x)(5+2x)$

(l) 36^2-6^2

Solution: $(36-6)(36+6)$ 30×42 1260

(m) $28^2-2.2^2$

Solution: $(28-2)(28+2)$ 26×30 780

(n) $3.214^2-2.214^2$

Solution: $214^2(3-2)$ 214^2 45796

(o) 7^2-6^2

Solution: $(7-6)(7+6)$ 1×13 13

(p) 101^2-100^2

Solution: $(101-100)(101+100)$ 1×201 201

7.3 Simple Quadratics

1. Factorise: 16

(a) $x^2+11x+18$

Solution: $(x+2)(x+9)$

(b) $x^2+11x+30$

Solution: $(x+5)(x+6)$

(c) $x^2+19x+90$

Solution: $(x+10)(x+9)$

(d) $x^2+7x+12$

Solution: $(x+3)(x+4)$

(e) $x^2+13x+40$

Solution: $(x+5)(x+8)$

(f) $x^2+28x+27$

Solution: $(x+1)(x+27)$

(g) $x^2+18x+56$

Solution: $(x+7)(x+8)$

(h) $x^2-14x+33$

Solution: $(x-3)(x-11)$

(i) $x^2-13x+42$

Solution: $(x-6)(x-7)$

(j) $x^2-47x+90$

Solution: $(x-2)(x-45)$

(k) $x^2-25x+100$

Solution: $(x-10)(x-15)$

(l) $x^2-21x+80$

Solution: $(x-5)(x-16)$

(m) $x^2-11x+24$

Solution: $(x-3)(x-8)$

(n) $x^2 - 14x + 24$

Solution: $(x-6)(x-8)$

(o) $x^2 - 8x - 33$

Solution: $(x-11)(x+3)$

(p) $x^2 - 19x - 42$

Solution: $(x-21)(x+2)$

2. Factorise: 16

(a) $x^2 - 9x - 90$

Solution: $(x-15)(x+6)$

(b) $x^2 + 15x - 100$

Solution: $(x-5)(x+20)$

(c) $x^2 - 7x - 60$

Solution: $(x-12)(x+5)$

(d) $x^2 - 10x - 24$

Solution: $(x-12)(x+2)$

(e) $x^2 + 2x - 15$

Solution: $(x-3)(x+5)$

(f) $x^2 + 5x - 24$

Solution: $(x-3)(x+8)$

(g) $x^2 + 8x + 12$

Solution: $(x+2)(x+6)$

(h) $x^2 + 11x + 30$

Solution: $(x+5)(x+6)$

(i) $x^2 - 9x - 90$

Solution: $(x-15)(x+6)$

(j) $x^2 - 7x - 18$

Solution: $(x-9)(x+2)$

(k) $x^2 + x - 90$

Solution: $(x-9)(x+10)$

(l) $x^2 + 14x + 49$

Solution: $(x+7)(x+7)$

(m) $x^2 - 18x + 81$

Solution: $(x-9)(x-9)$

(n) $x^2 + 12x + 36$

Solution: $(x+6)(x+6)$

(o) $x^2 - 16x + 64$

Solution: $(x-8)(x-8)$

(p) $x^2 - 8x + 16$

Solution: $(x-4)(x-4)$

7.4 Perfect Squares

1.

(a) $x^2 + 10x + 25$

Solution: $(x+5)(x+5)$

Solution: $(a + 14)(a + 14)$

Solution: $\left(x - \frac{11}{2}\right)\left(x - \frac{11}{2}\right)$

(b) $a^2 + 28a + 196$

(c) $x^2 - 11x + \frac{121}{4}$

2. Identify the simple quadratic expression that cannot be factorised as a perfect Square. 2

(a) $x^2 + \frac{2x}{3} + \frac{1}{9}$

(b) $x^2 - 3x + \frac{9}{4}$

(c) $x^2 - \frac{5x}{3} + \frac{25}{36}$

(d) $x^2 + \frac{7x}{4} + \frac{49}{16}$

(e) $x^2 - \frac{11x}{2} + \frac{121}{16}$

(f) $x^2 - \frac{4x}{5} + \frac{4}{25}$

(g) $x^2 - \frac{3x}{2} - \frac{9}{16}$

(h) $x^2 - \frac{9x}{2} + \frac{81}{16}$

Solution: $x^2 - \frac{3x}{2} - \frac{9}{16}$

7.5 Quadratics with Common Factors

1. Factorise: 16

(a) $3x^2 - 27x + 24$

Solution: $3(x - 1)(x - 8)$

(b) $5x^2 - 5x - 30$

Solution: $5(x - 3)(x + 2)$

(c) $5x^2 + 40x + 35$

Solution: $5(x + 7)(x + 1)$

(d) $5x^2 + 60x + 180$

Solution: $5(x + 6)(x + 6)$

(e) $3x^2 + 30x - 72$

Solution: $3(x - 4)(x + 6)$

(f) $3x^2 - 24x + 36$

Solution: $3(x - 4)(x - 3)$

(g) $3x^2 - 48$

Solution: $3(x - 4)(x + 4)$

(h) $3a^2 - 27b^2$

Solution: $3(a - 3b)(a + 3b)$

(i) $12 - 3m^2$

Solution: $3(2 - m)(2 + m)$

(j) $27x^2 - \frac{1}{3}y^2$

Solution: $\frac{1}{3}(9x - y)(9x + y)$

(k) $\frac{1}{4}x^2 - y^2$

Solution: $\left(\frac{1}{2}x - y\right)\left(\frac{1}{2}x + y\right)$

(l) $7 - 6x - x^2$

Solution: $-(x - 1)(x - 7)$

(m) $-x^2 - 14x - 45$

Solution: $-(x+5)(x+9)$

Solution: $-(x+7)(x+8)$

(n) $22x - x^2 - 40$

Solution: $-(x-8)(x-5)$

(o) $-56 - x^2 - 15x$

(p) $7x + 18 - x^2$

Solution: $-(x-2)(x-9)$

Question:	1	2	3	4	5	6	7	??	??	??	??	??	??	?
Points:	16	2	5	6	8	3	4	????	????	????	????	????	????	?
Score:														