Year 9 Mathematics
Topic 4
Factorisation

PEN Education

December 1, 2023

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Contents

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1 Introduction

Today we will cover the theory of **factorisation**. This is the process of taking something like $x^2 + 5x + 6$ and transforming it into $(x + 2)(x + 3)$. This looks familiar because it is the reverse process to *expansion*, which is what we learned in our **Topic 1** lesson.

1. Why do you think it is important to learn the factorisation technique? $\boxed{1}$

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2. What is a silly, real life example of factorisation? $\boxed{1}$

. . . .

Here is a box with a number of **buzz words** from today's class. As you come to understand these words, cross them off here.

common factor difference of two squares monic non-monic quadratic perfect square factorisation expansion

2 Common Factors

We can factorise expressions entirely by realising that there is a some term which can be pulled out of all the terms. Consider $4a + 12$. Here we have _____ common, and so this can be factorised as _____. Similarly, see for the following examples what term can be extracted from all the other ones.

Examples:

1. Factorise:

8

(a) $12x^2 + 3x$

. . . .

(b) $36ab - 27a$

. . . .

(c) $3x + 9$

. . . .

(d) $-7a^2 - 49$

. . . .

(e) $7a^2 + 63ab$

. . . .

(f) $5pq^2 + 10p^2q + 25p^2q^2$

. . . .

2.1 Exercises:

1. Complete each factorisation.

   (a) $12x = 12 \times$ _____

   (b) $24a = 12 \times$ _____

   (c) $15ac = 5c \times$ _____

   (d) $y^2 = y \times$ _____

   (e) $24a^2 = 6a \times$ _____

   (f) $-6b^2 = 2b \times$ _____

   (g) $4x^2y = 2x \times$ _____

   (h) $12m^2n = 3mn \times$ _____

2. Fill in the blanks by finding the missing factors.

   (a) $12a + 18 = 6\times$

   (b) $20mn - 15n = 5 \times$ _____

   (c) $a^2 + 4a = (a + 4) \times$ _____

   (d) $6yz^2 - 18yz = 3z \times$ _____

3. Factorise:

   (a) $6x + 24 =$ _____

   (b) $5a + 15 =$ _____

   (c) $y^2 + xy =$ _____

   (d) $4x + 24 =$ _____

   (e) $y^2 - 3y =$ _____

   (f) $-14a - 21 =$ _____

4. Factorise:

   (a) $4ab + 16a =$ _____

   (b) $12a^2 + 8a =$ _____

   (c) $15a^2b^2 + 10ab^2 =$ _____

   (d) $4a^2 + 6a =$ _____

   (e) $3b - 6b^2 =$ _____

   (f) $4x^2 - 6xy =$ _____

   (g) $18y - 9y^2 =$ _____

   (h) $4a - 6ab^2 =$ _____

   (i) $14mn^2 - 21m^2n =$ _____

   (j) $6pq^2 - 21qp^2 =$ _____

   (k) $-10b^2 + 5b =$ _____

   (l) $-4pq + 16p^2 =$ _____

   (m) $-16a^2b - 8ab =$ _____

   (n) $-8a^2b^2 - 2ab =$ _____

   (o) $-5x^2y + 30x =$ _____

   (p) $12xy^2 - 3x^2y =$ _____

5. In each part, an expression for the area of the rectangle has been given. Find an expression for the missing side length.

8

4

6

8

3

$2a + 3$	5	$6a$
Area $= 8a + 12$	Area $= 10b + 15$	Area $= 12a^2 + 6ab$
(a)	(b)	(c)

....
....
....
....

6. Factorise: 4

   (a) $4a^2b - 2ab + 8ab^2$ (c) $5a^2b + 3ab + 4ab^2$

   ....
   ....
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   (b) $7ab + 14a^2 + 21b$ (d) $5p^2q^2 + 10pq^2 + 15p^2q$

   ....
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3 Difference of Two Squares

Continueing with the theme of using our algebraic identities *backwards*, we have

$$(a + b)(a - b) = a^2 - b^2,$$

Which becomes:

$$a^2 - b^2 = (a + b)(a - b)$$

Now we can write expressions such as $x^2 - 9$ in the factored form of _____.

3.1 Examples:

1. Factorise: 14
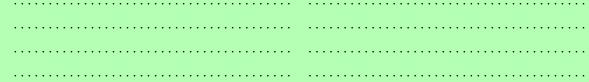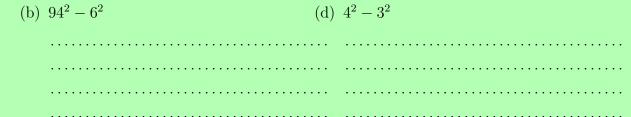
(a) $25 - y^2$

(h) $1 - 4a^2$

(b) $4x^2 - 9$

(i) $3x^2 - 48$

(c) $3a^2 - 27$

(j) $10x^2 - 1000$

(d) $-16 + 9x^2$

(k) $3 - 12b^2$

(e) $x^2 - 16$

(l) $45m^2 - 125n^2$

(f) $(2x)^2 - 25$

(m) $-25 + x^2$

(g) $9x^2 - 4$

(n) $-81x^2 + 16$

2. Use the factorisation of the difference of two squares to evaluate the following. One has been done for you.

$$17^2 - 3^2 = (17 + 3)(17 - 3)$$
$$= 20 \times 14$$
$$= 280$$

5

(a) $23^2 - 7^2$

(c) $11.3^2 - 8.7^2$

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(b) $94^2 - 6^2$

(d) $4^2 - 3^2$

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Note: You can always check your factorisation is correct by expanding your result and checking to see if that is the expression you started with!

3.2 Exercises:

1. Factorise:

16

(a) $x^2 - 49$

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(g) $100a^2 - 49b^2$

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(b) $(3x)^2 - 16$

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(h) $25a^2 - 100b^2$ d $d^2 - 400$

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....

(c) $16y^2 - 49$

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....

(i) $4x^2 - 100$

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....

(d) $9 - 16y^2$

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....

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....

(j) $5x^2 - 45$

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(e) $a^2 - 121$

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....

(k) $7x^2 - 63$

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....

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....

(f) $(4x)^2 - 1$

....

....

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....

(l) $8x^2 - 50$

....

....

....

....

(m) $20 - 5y^2$

(t) $23^2 - 3^2$

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(n) $27a^2 - 12b^2$

(u) $1.8^2 - 0.2^2$

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(o) $27a^2 - 192l^2$

(v) $92.6^2 - 7.4^2$

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(p) $-8x^2 + 32y^2$

(w) $5^2 - 4^2$

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(q) $-4 + 9x^2$

(x) $9^2 - 8^2$

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(r) $-100x^2 + 9$

(y) $6^2 - 5^2$

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(s) $-36x^2 + 400$

(z) $10^2 - 9^2$

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4 Simple Quadratics

The nomenclature "simple" quadratic means that the coefficient of x^2 is just 1. Let us recall some definitions:
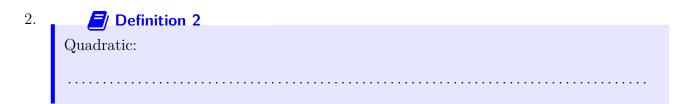
1. **📘 Definition 1** $\boxed{1}$

   Nomenclature:

   ....

2. **📘 Definition 2** $\boxed{1}$

   Quadratic:

   ....

3. **📘 Definition 3** $\boxed{1}$

   Coefficient:

   ....

The Technique:
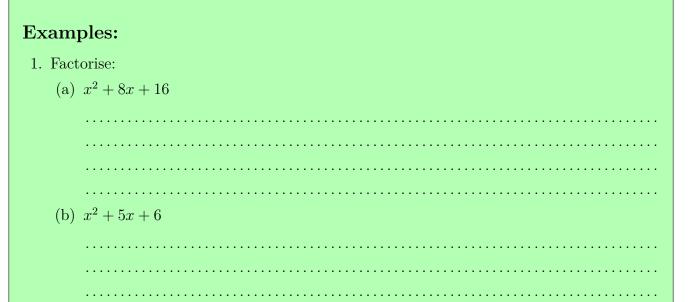
Let us take a concrete example: $x^2 - 3x - 18$. To factorise this monic quadratic we need to find two numbers that multiply to give -18 and add to give -3. Such numbers will be _____ and _____. Thus we can write $x^2 - 3x - 18 = (x - \ldots)(x + \ldots)$

Examples:

1. Factorise: $\boxed{2}$

   (a) $x^2 + 8x + 16$

   ....

   ....

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   ....

   (b) $x^2 + 5x + 6$

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   (c) $x^2 + 7x + 10$

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(d) $x^2 + 9x + 14$

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Exercises:

1. Factorise:

(a) $x^2 + 9x + 20$

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(b) $x^2 + 12x + 32$

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(c) $x^2 + 20x + 75$

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(d) $x^2 + 15x + 56$

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(e) $x^2 - 5x + 6$

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(f) $x^2 - 17x + 30$

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(g) $x^2 - 9x + 14$

$\dotfill$

$\dotfill$

$\dotfill$

$\dotfill$

(h) $x^2 - 15x + 44$

$\dotfill$

$\dotfill$

$\dotfill$

$\dotfill$

(i) $x^2 - 18x + 80$

$\dotfill$

$\dotfill$

$\dotfill$

$\dotfill$

(j) $x^2 - 14x + 40$

$\dotfill$

$\dotfill$

$\dotfill$

$\dotfill$

(k) $x^2 - 30x + 56$

$\dotfill$

$\dotfill$

$\dotfill$

$\dotfill$

(l) $x^2 + x - 6$

$\dotfill$

$\dotfill$

$\dotfill$

$\dotfill$

(m) $x^2 + x - 30$

$\dotfill$

$\dotfill$

$\dotfill$

$\dotfill$

(n) $x^2 - 5x - 14$

$\dotfill$

$\dotfill$

$\dotfill$

$\dotfill$

(o) $x^2 - 7x - 44$

$\dotfill$

$\dotfill$

$\dotfill$

$\dotfill$

(p) $x^2 + 2x - 80$

$\dotfill$

$\dotfill$

$\dotfill$

$\dotfill$

(q) $x^2 + 3x - 40$

$\dotfill$

$\dotfill$

$\dotfill$

$\dotfill$

(r) $x^2 + 4x - 21$

$\dotfill$

$\dotfill$

$\dotfill$

$\dotfill$

(m) $x^2 - x + 56$

(q) $x^2 - 5x + 4$

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

(n) $x^2 - 3x + 2$

(r) $x^2 - x - 12$

. . . .

. . . .

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. . . .

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(o) $x^2 - 3x - 10$

(s) $x^2 + 3x - 10$

. . . .

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(p) $x^2 - 5x - 14$

(t) $x^2 + 6x + 9$

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5   Perfect Squares

The expansion of a perfect square has a special form. For example:

$$
\begin{aligned}
(x + 3)^2 &= (x + 3)(x + 3) \\
&= x^2 + 6x + 9 \\
&= x^2 + 2 \times (3x) + 3^2
\end{aligned}
$$

5.1   Theorem

$$
a^2 + 2ab + b^2 = (a + b)^2 \tag{1}
$$
$$
a^2 - 2ab + b^2 = (a - b)^2 \tag{2}
$$

Examples:

1. (a) $x^2 + 8x + 16$

3

. . . .

. . . .

. . . .

. . . .

(b) $x^2 - 10x + 25$

. . . .

. . . .

. . . .

. . . .

(c) $x^2 + 11x + \frac{121}{4}$

. . . .

. . . .

. . . .

. . . .

2. Identify the simple quadratic expression that cannot be factorised as a perfect square.

2

(a) $x^2 + 4x + 4$

(e) $x^2 + 6x + 9$

. . . .

. . . .

(b) $x^2 - 6x + 12$

(f) $x^2 + 5x + \frac{25}{4}$

. . . .

. . . .

(c) $x^2 - 12x + 36$

(g) $x^2 - 8x - 16$

. . . .

. . . .

(d) $x^2 - 10x + 25$

(h) $x^2 - 14x + 49$

. . . .

. . . .

Exercises:

1. (a) $x^2 + 12x + 36$

....

....

(b) $a^2 - 4a + 4$

....

....

(c) $x^2 - 9x + \frac{81}{4}$

....

....

(d) $x^2 + 8x + 16 = (x + \ldots)^{\cdots}$

....

....

(e) $x^2 - \ldots + \ldots = (x - 9)^2$

....

....

(f) $x^2 - 8x + 16$

....

....

(g) $m^2 - 26m + 169$

....

....

(h) $x^2 + 13x + \frac{169}{4}$

....

....

$\boxed{8}$

2. A brick company provides rectangular and square pavers.

$\boxed{4}$

(a) Draw a diagram to show how two different square pavers of side lengths a and b respectively, and two identical rectangular pavers with dimensions $a \times b$, can be arranged into a square.

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(b) How many of each type of paver enables you to pave a square area of side length $a + 3b$? Draw a diagram to illustrate how this can be done.

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6 Quadratics with Common Factors

The quadratics for this section are a little less tame. The x^2 term now possesses a coefficient which we will be trying to pull out of the entire expression to use our techniques from section 4 on the new expression within the parenthesis.

Examples:

1. Factorise:

    (a) $3x^2 + 9x + 6$

    ....

    ....

    ....

    ....

    (b) $6x^2 - 54$

    ....

    ....

    ....

    ....

    (c) $-x^2 - x + 2$

    ....

    ....

    ....

    ....

3

2. (a) $2x^2 + 14x + 24$

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(b) $4x^2 - 24x + 36$

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(c) $4x^2 - 4x + 48$

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(d) $3x^2 + 9x - 120$

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(e) $2x^2 - 4x - 96$

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(f) $3x^2 - 18x + 27$

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....

....

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(g) $4x^2 - 16$

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....

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(h) $3a^2 - 27$

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(i) $27x^2 - 3y^2$

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....

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(j) $128 - 2x^2$

....

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....

(k) $\frac{1}{4}a^2 - 9$

(n) $-x^2 + 3x + 40$

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....

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....

(l) $-x^2 - 8x - 12$

(o) $11x - x^2 - 24$

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(m) $9 + 8x - x^2$

(p) $-16x - 63 - x^2$

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Exercises:

1. Factorise:

16

(a) $3x^2 + 24x + 36$

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(b) $7x^2 + 14x + 7$

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(c) $2x^2 - 18x + 36$

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....

(d) $3x^2 - 3x - 90$

....

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(e) $5x^2 + 65x + 180$

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(f) $5x^2 - 20x + 20$

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(g) $2x^2 - 18$

....

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(h) $6x^2 - 600$

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(i) $45 - 5b^2$

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(j) $\frac{1}{2}a^2 - 2b^2$

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(k) $\frac{1}{5}x^2 - 20$

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(l) $12 - 11x - x^2$

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(a) $-x^2 - 4x - 4$

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(c) $-3x^2 - 30x + 72$

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(b) $42 + x - x^2$

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....

(d) $-x^2 - 35 + 12x$

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7 Homework

7.1 Common Factors

1. Fill in the gaps:

   (a) $36ab = 9a \times$ _____

   (b) $6y^2 = 3y \times$ _____

   (c) $8a^2b = 2ab \times$ _____

   (d) $25a^2b^2 = 5ab \times$ _____

   $\boxed{4}$

2. Factorise:

   (a) $16ab + 10b^2 - 2a^2b$

   (b) $2x^2 + 4x$

   $\boxed{2}$

3. Fill in the blanks:

   (a) $15p - 10 = 5 \times$ _____

   (b) $20mn - 15n = 5n \times$ _____

   (c) $b^2 - 10b = b \times$ _____

   (d) $6yz^2 - 18yz = yz \times$ _____

   (e) $6yz^2 - 18yz = 6yz \times$ _____

   $\boxed{5}$

4. Pull out the common factor:

   (a) $ac + 5c$

   (c) $7a - 63$

   (e) $-6y - 9$

   ....

   (b) $a^2 + a$

   (d) $9a + 36$

   (f) $-4 - 12b$

   ....

   $\boxed{6}$

5. Factorise:

   (a) $18m^2n + 9mn^2$ (c) $9mn - 12m^2n$ (e) $10ab^2 - 25a^2b$ (g) $18p^2 - 4pq$

   ....

   (b) $8a^2 + 12ab$ (d) $6xy - 4x^2$ (f) $-x^2y - 3xy$ (h) $-25m^2n^2 - 10mn^2$

   ....

   $\boxed{8}$

6. In each part, an expression for the area of the rectangle has been given. Find an expression for the missing side length.

   $\boxed{3}$

   $3a$

   Area $= 9a + 6ab$

   (a)

   ....
   ....
   ....(b)
   ....

   a

   Area $= 2a + a^2$

$$3q^2 - 2$$

.... | Area $= 24p^2q^2 - 16p^2$ | ....
.... | | ....
.... | | ....
.... (c)   ....

7. Factorise: $\boxed{4}$

   (a) $4m^2n - 4mn + 16n^2$ (c) $6a + 8ab + 10ab^2$

    ....
    ....
    ....

   (b) $2m^2 + 4mn + 6n$ (d) $5\ell^2 - 15\ell m - 20m^2$

    ....
    ....
    ....

7.2 Difference of Two Squares

1. Factorise: $\boxed{16}$

   (a) $d^2 - 400$

    ....
    ....
    ....

   (b) $(5m)^2 - 9$ (f) $12m^2 - 75$

    ....
    ....
    ....

   (c) $64m^2 - 81p^2$ (g) $16x^2 - 100y^2$

    ....
    ....
    ....

   (d) $-9 + x^2$ (h) $-200p^2 + 32q^2$

    ....
    ....
    ....

   (e) $6x^2 - 24$ (i) $-9x^2 + 4$

(m) $28^2 - 2.2^2$

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(j) $-18 + 50x^2$

(n) $3.214^2 - 2.214^2$

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....

(k) $-175 + 28x^2$

(o) $7^2 - 6^2$

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(l) $36^2 - 6^2$

(p) $101^2 - 100^2$

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7.3 Simple Quadratics

1. Factorise: $\boxed{16}$

(a) $x^2 + 11x + 18$

(e) $x^2 + 13x + 40$

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(b) $x^2 + 11x + 30$

(f) $x^2 + 28x + 27$

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....

(c) $x^2 + 19x + 90$

(g) $x^2 + 18x + 56$

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....

....

(d) $x^2 + 7x + 12$

(h) $x^2 - 14x + 33$

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....

(i) $x^2 - 13x + 42$

(m) $x^2 - 11x + 24$

$\dotfill$

$\dotfill$

$\dotfill$

(j) $x^2 - 47x + 90$

(n) $x^2 - 14x + 24$

$\dotfill$

$\dotfill$

$\dotfill$

(k) $x^2 - 25x + 100$

(o) $x^2 - 8x - 33$

$\dotfill$

$\dotfill$

$\dotfill$

(l) $x^2 - 21x + 80$

(p) $x^2 - 19x - 42$

$\dotfill$

$\dotfill$

$\dotfill$

2. Factorise:

16

(a) $x^2 - 9x - 90$

$\dotfill$

$\dotfill$

$\dotfill$

(b) $x^2 + 15x - 100$

(f) $x^2 + 5x - 24$

$\dotfill$

$\dotfill$

$\dotfill$

(c) $x^2 - 7x - 60$

(g) $x^2 + 8x + 12$

$\dotfill$

$\dotfill$

$\dotfill$

(d) $x^2 - 10x - 24$

(h) $x^2 + 11x + 30$

$\dotfill$

$\dotfill$

$\dotfill$

(e) $x^2 + 2x - 15$

(i) $x^2 - 9x - 90$

(m) $x^2 - 18x + 81$

....
....
....

(j) $x^2 - 7x - 18$ (n) $x^2 + 12x + 36$

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....
....

(k) $x^2 + x - 90$ (o) $x^2 - 16x + 64$

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....
....

(l) $x^2 + 14x + 49$ (p) $x^2 - 8x + 16$

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....
....

7.4 Perfect Squares

1. (a) $x^2 + 10x + 25$ (b) $a^2 + 28a + 196$ (c) $x^2 - 11x + \frac{121}{4}$ $\boxed{3}$

   ....
   ....
   ....

2. Identify the simple quadratic expression that cannot be factorised as a perfect Square. $\boxed{2}$

   (a) $x^2 + \frac{2x}{3} + \frac{1}{9}$ (e) $x^2 - \frac{11x}{2} + \frac{121}{16}$
   (b) $x^2 - 3x + \frac{9}{4}$ (f) $x^2 - \frac{4x}{5} + \frac{4}{25}$
   (c) $x^2 - \frac{5x}{3} + \frac{25}{36}$ (g) $x^2 - \frac{3x}{2} - \frac{9}{16}$
   (d) $x^2 + \frac{7x}{4} + \frac{49}{16}$ (h) $x^2 - \frac{9x}{2} + \frac{81}{16}$

7.5 Quadratics with Common Factors

1. Factorise: $\boxed{16}$

   (a) $3x^2 - 27x + 24$ (b) $5x^2 - 5x - 30$

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   ....
   ....

(c) $5x^2 + 40x + 35$

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(d) $5x^2 + 60x + 180$

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(e) $3x^2 + 30x - 72$

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(f) $3x^2 - 24x + 36$

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(g) $3x^2 - 48$

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(h) $3a^2 - 27b^2$

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(i) $12 - 3m^2$

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(j) $27x^2 - \frac{1}{3}y^2$

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(k) $\frac{1}{4}x^2 - y^2$

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(l) $7 - 6x - x^2$

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(m) $-x^2 - 14x - 45$

....

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(n) $22x - x^2 - 40$

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(o) $-56 - x^2 - 15x$

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(p) $7x + 18 - x^2$

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Question:	1	2	3	4	5	6	7	??	??	??	??	??	??	
Points:	16	2	5	6	8	3	4	????	????	????	????	????	????	??
Score:														