Year 9 Mathematics
Topic 6: Formulas

PEN Education

2024

Contents

1 Introduction

formula subject pronumeral substitution expression construction equation rearrange

1. What is a formula? [1]

   Solution: A formula is an expression that which relates different quantities.

2. What is the most famous formula that you know? [4]

Solution:

$$E = mc^2$$

(a) Can you derive this formula?

Solution:

(b) What is the subject of your formula?

Solution: E

(c) What do all of the variables stand for?

Solution: In the formula $E = mc^2$:

- E stands for energy.
- m stands for mass.
- c stands for the speed of light in a vacuum.

2 Substitution into formulas

This section is straight-forward: you substitute the unknown values for the values that the question gives you. We are practising this so that you can do it accurately and quickly.

1. Find the value of the subject when the pronumerals in the formula have the values indicated. 2

(a) $F = ma$, where $a = 10, m = 3.5$

Solution:

$$\begin{aligned} F &= ma \\ &= 10 \times 3.5 \\ &= 35 \end{aligned}$$

(b) $m = \frac{a+b}{2}$, where $a = 12, b = 26$

Solution:

$$m = \frac{a+b}{2}$$
$$= \frac{12+26}{2}$$
$$= 19$$

2. The formula for the circumference C of a circle of radius r is $C = 2\pi r$. Find the value of C when $r = 20$: $\boxed{2}$

   (a) in terms of π (that is, exactly)

   Solution: $C = 2\pi r = 40\pi$

   (b) correct to 2 decimal places

   Solution: $C = 40\pi = 125.663\ldots$ (using a calculator) ≈ 125.66 (correct to 2 decimal places)

3. (a) The area of a triangle A cm^2 is given by $A = \frac{1}{2}bh$, where b cm is the base length and h cm is the height. Calculate the area of a triangle with base length 16 cm and height 11 cm. $\boxed{2}$

   Solution:

   $$A = \frac{1}{2}bh$$
   $$= \frac{1}{2} \times 16 \times 11$$
   $$= 88$$

   The area of the triangle is 88 cm^2.

   (b) The simple interest payable when \$$P$ is invested at a rate of r% per year for t years is given by $I = \frac{Prt}{100}$. Calculate the simple interest payable when \$1000 is invested at 3.5% per year for 6 years. $\boxed{2}$

   Solution:

   $$I = \frac{Prt}{100}$$
   $$= \frac{1000 \times 3.5 \times 6}{100}$$
   $$= 210$$

   The interest payable is \$210.

4. For a car travelling in a straight line with initial velocity u m/s and acceleration a m/s^2, the formula for the velocity v m/s at time t seconds is $v = u + at$. $\boxed{2}$

   (a) Find u if $a = 2, v = 15$ and $t = 7$.

   Solution: $v = u + at$ When $a = 2, v = 15$ and $t = 7$.

$$\begin{aligned} 15 &= u + 2 \times 7 \\ 15 &= u + 14 \\ u &= 1 \end{aligned}$$

   The initial velocity is 1 m/s.

   (b) Find a if $v = 10, u = 6$ and $t = 3$.

   Solution: $v = u + at$ When $v = 10, u = 6$ and $t = 3$.

$$\begin{aligned} 10 &= 6 + 3a \\ 4 &= 3a \\ a &= \frac{4}{3} \end{aligned}$$

   The acceleration is $\frac{4}{3}$ m/s^2.

5. The thin lens formula states that $\boxed{2}$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

   where u is the distance from the object to the lens, v is the distance of the image from the lens and f is the focal length of the lens.

   (a) Find f if $u = 2$ and $v = 5$.

   Solution: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$
   When $u = 2$ and $v = 5$,
   $\frac{1}{f} = \frac{1}{2} + \frac{1}{5}$

$$= \frac{7}{10}$$

   Taking reciprocals of both sides
   of the equation gives $f = \frac{10}{7}$.

   (b) Find u if $f = 2$ and $v = 6$.

Solution: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ If $f = 2$ and $v = 6$,
$\frac{1}{2} = \frac{1}{u} + \frac{1}{6}$
$\frac{1}{u} = \frac{1}{2} - \frac{1}{6}$
$= \frac{1}{3}$
Taking reciprocals of both sides of the equation gives $u = 3$.

6. The area of a circle A cm^2 is given by $A = \pi r^2$, where r cm is the radius of the circle. If $A = 20$, find r : 2

   (a) exactly

   Solution: $A = \pi r^2$
   When $A = 20$,
   $20 = \pi r^2$
   $\frac{20}{\pi} = r^2$
   (Divide both sides of equation by π.)
   $r = \sqrt{\frac{20}{\pi}}$ (r is positive.)

   (b) correct to 2 decimal places

   Solution: $r \approx 2.52$ (correct to 2 decimal places)

1. For each part, find the value of the subject when the other pronumerals have the value indicated. 3

   (a) $A = \ell w$, where $\ell = 5, w = 8$

   Solution: $A = 5 \times 8 = 40$

   (b) $s = \frac{d}{t}$, where $d = 120, t = 6$

   Solution: $s = \frac{120}{6} = 20$

   (c) $A = \frac{1}{2}xy$, where $x = 10, y = 7$

   Solution: $A = \frac{1}{2} \times 10 \times 7 = 35$

2. For the formula $v = u + at$, find: 4

(a) v if $u = 6, a = 3$ and $t = 5$

Solution: $v = 6 + 3 \times 5 = 21$

(b) u if $v = 40, a = 5$ and $t = 2$

Solution: $u = 40 - 5 \times 2 = 30$

(c) a if $v = 60, u = 0$ and $t = 5$

Solution: $a = \frac{60-0}{5} = 12$

(d) t if $v = 100, u = 20$ and $a = 6$

Solution: $t = \frac{100-20}{6} \approx 13.33$

3. $\boxed{4}$

(a) For the formula $S = 2(a - b)$, find a if $S = 60$ and $b = 10$.

Solution: $a = \frac{60}{2} + 10 = 40$

(b) For the formula $I = \frac{180n-360}{n}$, find n if $I = 120$.

Solution: $180n - 360 = 120n$ $60n = 360$ $n = 6$

(c) For the formula $a = \frac{m+n}{2}$, find m if $a = 20$ and $n = 6$.

Solution: $m = 2 \times 20 - 6 = 34$

(d) For the formula $A = \frac{PRT}{100}$, find P if $A = 1600, R = 4$ and $T = 10$.

Solution: $P = \frac{1600 \times 100}{4 \times 10} = 4000$

4. For the formula $s = ut + \frac{1}{2}at^2$, find the value of: $\boxed{3}$

(a) u, when $s = 10, t = 20$ and $a = 2$

Solution: $10 = u \times 20 + \frac{1}{2} \times 2 \times 20^2$ $10 = 20u + 400$ $u = -19.5$

(b) a, when $s = 20, u = 5$ and $t = 2$

Solution: $20 = 5 \times 2 + \frac{1}{2} \times a \times 2^2$ $20 = 10 + 2a$ $a = 5$

5. Given that $P = \frac{M+m}{M-m}$, find the value of P when: $\boxed{3}$

(a) $M = 8$ and $m = 4$

Solution: $P = \frac{8+4}{8-4} = 3$

(b) $M = 26$ and $m = 17$

Solution: $P = \frac{26+17}{26-17} = \frac{43}{9}$

6. The area A cm^2 of a square with side length x cm is given by $A = x^2$. If $A = 20$, find: 3

   (a) the value of x

   Solution: $x = \sqrt{20} \approx 4.47$

   (b) the value of x correct to 2 decimal places.

   Solution: $x \approx 4.47$ (to 2 decimal places)

7. For a rectangle of length ℓ cm and width w cm, the perimeter P cm i given by $P = 2(\ell+w)$. Use this formula to calculate the length of a rectangle which has width 15 cm and perimeter 57 cm. 2

   Solution: $57 = 2(\ell + 15)$ $\ell = \frac{57}{2} - 15 = 13.5$

8. The area A cm^2 of a triangle with side lengths, a cm, b cm and c cm is given by *Heron's formula*: 6

   ## Heron's Formula

   $$A^2 = s(s-a)(s-b)(s-c)$$

   where $s = \frac{a+b+c}{2}$ = half the perimeter. This is helpful for finding the area of non-right-angled triangles for whic you do not even have an angle for. Try to determine the area of this shape yourself without the formula if you dare.

   Find the exact areas of the triangles whose side lengths are given below.

   (a) 6 cm, 8 cm and 10 cm

   Solution: $s = \frac{6+8+10}{2} = 12$ $A = \sqrt{12(12-6)(12-8)(12-10)} = \sqrt{12 \times 6 \times 4 \times 2} = \sqrt{576} = 24$

   (b) 5 cm, 12 cm and 13 cm

   Solution: $s = \frac{5+12+13}{2} = 15$ $A = \sqrt{15(15-5)(15-12)(15-13)} = \sqrt{15 \times 10 \times 3 \times 2} = \sqrt{900} = 30$

   (c) 8 cm, 10 cm and 14 cm

Solution: $s = \frac{8+10+14}{2} = 16$ $A = \sqrt{16(16-8)(16-10)(16-14)} = \sqrt{16 \times 8 \times 6 \times 2} = \sqrt{1536} \approx 39.19$

(d) 13 cm, 14 cm and 15 cm

Solution: $s = \frac{13+14+15}{2} = 21$ $A = \sqrt{21(21-13)(21-14)(21-15)} = \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84$

3 Changing the subject of a formula

Up until now it has been trivial to substitute values into *formulas* and have the answer pop out quite simply. But now we shall need to do some work to find the answers - we will need to **rearrange** the **expression**.

Definition 1

Trivial:

Solution: Something that is easy, simple or self-evident.

1. The manager of a bed-and-breakfast guest house finds that the weekly profit $\$P$ is given by the formula 1

$$P = 40G - 600$$

where G is the number of guests who stay during the week. Make G the subject of the formula and use the result to find the number of guests needed to make a profit of \$800.

Solution:

$$\begin{aligned} P &= 40G - 600 \\ P + 600 &= 40G \\ G &= \frac{P+600}{40} \\ \text{When } P &= 800, \\ G &= \frac{800+600}{40} \\ &= \frac{1400}{40} = 35 \end{aligned}$$

Thirty-five guests are required to make a profit of \$800.

2. Given the formula $v^2 = u^2 + 2as$: 3

(a) rearrange the formula to make s the subject

Solution: $v^2 = u^2 + 2as$

$$\begin{aligned} v^2 - u^2 &= 2as && \text{(Subtract } u^2 \text{ from both sides o} \\ s &= \tfrac{v^2-u^2}{2a} && \text{(Divide both sides of the equation by } 2a.) \end{aligned}$$

(b) find the value of s when $u = 4, v = 10$ and $a = 2$

Solution: When $u = 4, v = 10$ and $a = 2$.

$$\begin{aligned} s &= \frac{10^2 - 4^2}{2 \times 2} \\ &= \frac{100 - 16}{4} \\ &= \frac{84}{4} \\ &= 21 \end{aligned}$$

(c) find the value of s when $u = 4, v = 12$ and $a = 3$

Solution: When $u = 4, v = 12$ and $a = 3$.

$$\begin{aligned} s &= \frac{12^2 - 4^2}{2 \times 3} \\ &= \frac{144 - 16}{6} \\ &= \frac{128}{6} \\ &= 21\frac{1}{3} \end{aligned}$$

3. Rearrange each of these formulas to make the pronumeral in brackets the subject. $\boxed{4}$

(a) $E = \frac{p^2}{2m}$ (m)

Solution: $E = \frac{p^2}{2m}$

$$\begin{aligned} mE &= \frac{p^2}{2} && \text{(Multiply both sides of the equation by } m.) \\ m &= \frac{p^2}{2E} && \text{(Divide both sides by } E.) \end{aligned}$$

(b) $T = 2\pi\sqrt{\frac{\ell}{g}}$ (ℓ)

Solution:

$$\begin{aligned} T &= 2\pi\sqrt{\tfrac{\ell}{g}} \\ \tfrac{T}{2\pi} &= \sqrt{\tfrac{\ell}{g}} \quad \text{(Divide both sides of the equation by } 2\pi.\text{)} \\ \tfrac{\ell}{g} &= \left(\tfrac{T}{2\pi}\right)^2 \quad \text{(Square both sides of the equat} \\ &= \tfrac{T^2}{4\pi^2} \\ \ell &= \tfrac{T^2}{4\pi^2} \times g \quad \text{(Multiply both sides of the equation by } g.\text{)} \end{aligned}$$

That is, $\ell = \frac{T^2 g}{4\pi^2}$

(c) $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ (u)

Solution: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ $\frac{1}{f} - \frac{1}{v} = \frac{1}{u}$ (Subtract $\frac{1}{v}$ from both sides.) $\frac{v-f}{fv} = \frac{1}{u}$ (common denominator on LHS of equation)

$$u = \frac{fv}{v-f}$$

(Take reciprocals of both sides.)

Note: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ does not imply $f = u + v$.

(d) $P = \sqrt{h+c} - a$ (h)

Solution:

$$P = \sqrt{h+c} - a$$

$$\begin{aligned} P + a &= \sqrt{h+c} \quad \text{(Add } a \text{ to both sides of the equation.)} \\ (P+a)^2 &= h + c \quad \text{(Square both sides.)} \\ h &= (P+a)^2 - c \end{aligned}$$

The previous example shows some of the techniques that can be used to rearrange a formula.

(e) $\frac{3p}{4} - \frac{5}{q} = \frac{p^2}{3q}$ (q)

Solution:

$$\frac{3p}{4} - \frac{5}{q} = \frac{p^2}{3q}$$

$$12q\left(\frac{3p}{4} - \frac{5}{q}\right) = 12q \times \frac{p^2}{3q}$$

$3p \times 3q - 5 \times 12 = p^2 \times 4$ (Multiply both sides by the lowest common denominator, $12q$.)

$$9pq - 60 = 4p^2$$

$$9pq = 4p^2 + 60$$

$$q = \frac{4p^2 + 60}{9p}$$

1. The profit P made each day by a store owner who sells CDs is given by the formula $P = 5n - 150$, where n is the number of CDs sold.

   (a) What profit is made if the store owner sells 60 CDs? 1

   Solution: $P = 5n - 150$
   $P = 5(60) - 150$
   $P = 300 - 150$
   $P = \$150$

   (b) Make n the subject of the formula. 1

   Solution: $P = 5n - 150$
   $P + 150 = 5n$
   $n = \frac{P+150}{5}$

   (c) How many CDs were sold if the store made: 2

   i. a profit of \$275 ?

   Solution: $n = \frac{P+150}{5}$
   $n = \frac{275+150}{5}$
   $n = \frac{425}{5}$
   $n = 85$

   ii. a profit of \$400 ?

   Solution: $n = \frac{P+150}{5}$
   $n = \frac{400+150}{5}$
   $n = \frac{550}{5}$
   $n = 110$

iii. a loss of \$100 ?

Solution: $n = \frac{P+150}{5}$
$n = \frac{-100+150}{5}$
$n = \frac{50}{5}$
$n = 10$

iv. no profit?

Solution: $n = \frac{P+150}{5}$
$n = \frac{0+150}{5}$
$n = \frac{150}{5}$
$n = 30$

2. The cost C of hiring a reception room for a function is given by the formula $C = n + 250$, where n is the number of people attending the function.

(a) Rearrange the formula to make n the subject. 1

Solution: $C = 12n + 250$
$C - 250 = 12n$
$n = \frac{C-250}{12}$

(b) How many people attended the function if the cost of hiring the reception room was: 2

i. \$730 ?

Solution: $n = \frac{C-250}{12}$
$n = \frac{730-250}{12}$
$n = \frac{480}{12}$
$n = 40$

ii. \$1090 ?

Solution: $n = \frac{C-250}{12}$
$n = \frac{1090-250}{12}$
$n = \frac{840}{12}$
$n = 70$

iii. \$1210 ?

Solution: $n = \frac{C-250}{12}$
$n = \frac{1210-250}{12}$
$n = \frac{960}{12}$
$n = 80$

iv. \$1690 ?

Solution: $n = \frac{C-250}{12}$
$n = \frac{1690-250}{12}$
$n = \frac{1440}{12}$
$n = 120$

3. Given the formula $t = a + (n-1)d$:
   (a) rearrange the formula to make a the subject 1

   Solution: $t = a + (n-1)d$
   $a = t - (n-1)d$

   (b) find the value of a when: 2
      i. $t = 11, n = 4$ and $d = 3$

      Solution: $a = t - (n-1)d$
      $a = 11 - (4-1) \cdot 3$
      $a = 11 - 3 \cdot 3$
      $a = 11 - 9$
      $a = 2$

      ii. $t = 8, n = 5$ and $d = -3$

      Solution: $a = t - (n-1)d$
      $a = 8 - (5-1) \cdot (-3)$
      $a = 8 - 4 \cdot (-3)$
      $a = 8 + 12$
      $a = 20$

   (c) rearrange the formula to make d the subject 1

   Solution: $t = a + (n-1)d$
   $d = \frac{t-a}{n-1}$

   (d) find the value of d when: 2
      i. $t = 48, a = 3$ and $n = 16$

      Solution: $d = \frac{t-a}{n-1}$
      $d = \frac{48-3}{16-1}$
      $d = \frac{45}{15}$
      $d = 3$

      ii. $t = 120, a = -30$ and $n = 101$

Solution: $d = \frac{t-a}{n-1}$
$d = \frac{120-(-30)}{101-1}$
$d = \frac{150}{100}$
$d = 1.5$

(e) rearrange the formula to make n the subject and find the value of n when $t = 150, a = 5$ and $d = 5$ 1

Solution: $t = a + (n-1)d$
$n = \frac{t-a}{d} + 1$
$n = \frac{150-5}{5} + 1$
$n = \frac{145}{5} + 1$
$n = 29 + 1$
$n = 30$

4. Rearrange each of these formulas to make the pronumeral in brackets the subject.

(a) $y = mx + c$ (c)

Solution: $c = y - mx$

(b) $A = \frac{1}{2}bh$ (x)

Solution: $b = \frac{2A}{h}$

(c) $P = A + 2\ell h$ (ℓ)

Solution: $\ell = \frac{P-A}{2h}$

(d) $A = 2\pi r^2 + 2\pi rh$ (h)

Solution: $h = \frac{A-2\pi r^2}{2\pi r}$

(e) $s = \frac{n}{2}(a + \ell)$ (a)

Solution: $a = \frac{2s}{n} - \ell$

(f) $V = \pi r^2 + \pi rs$ (s)

Solution: $s = \frac{V}{\pi r} - r$

5. The formula for the sum S of the interior angles in a convex n-sided polygon $S = 180(n-2)$. Rearrange the formula to make n the subject and use this to find the number of sides in the polygon if the sum of the interior angles is: 3

(a) 1080° (b) 1800° (c) 3240°

Solution: $n = \frac{S}{180} + 2$
$n = \frac{1080}{180} + 2$
$n = 6 + 2$
$n = 8$

Solution: $n = \frac{S}{180} + 2$
$n = \frac{1800}{180} + 2$
$n = 10 + 2$
$n = 12$

Solution: $n = \frac{S}{180} + 2$
$n = \frac{3240}{180} + 2$
$n = 18 + 2$
$n = 20$

6. 8 When an object is shot up into the air with a speed of u metres per second, its height above the ground h metres and time of flight t seconds are related (ignoring air resistance by $h = ut - 4.9t^2$. 2

Find the speed at which an object was fired if it reached a height of 27.5 metres after 5 seconds.

Solution: $h = ut - 4.9t^2$
$27.5 = u(5) - 4.9(5)^2$
$27.5 = 5u - 4.9(25)$
$27.5 = 5u - 122.5$
$5u = 27.5 + 122.5$
$5u = 150$
$u = \frac{150}{5}$
$u = 30$ metres per second

7. Rearrange each of these formulas to make the pronumeral in brackets the subject. (All pronumerals represent positive numbers.) 4

(a) $c = a^2 + b^2$ (a)

Solution: $a = \sqrt{c - b^2}$

(b) $x = \sqrt{ab}$ (b)

Solution: $b = \frac{x^2}{a}$

(c) $T = \frac{2\pi}{n}$ (n)

Solution: $n = \frac{2\pi}{T}$

(d) $E = \frac{m}{2r^2}$ (r)

Solution: $r = \sqrt{\frac{m}{2E}}$

4 Constructing Formulas

This part is the most challenging yet. It requires a degree of conceptual thought. Practise as always will carve this skill groove deeper within your mind, but be prepared to get things incorrect in this section.

1. Find a formula for n, the number of cents in x dollars and y cents. 1

   Solution: In \$$x$ there are $100x$ cents.
   In \$$x$ and y cents there are $(100x + y)$ cents.
   The formula is $n = 100x + y$.

2. Here is an isosceles triangle with equal base angles marked. Find a formula for β in terms of α. 2

   Solution:
$$\begin{aligned} \alpha + 2\beta &= 180 \qquad \text{(angle sum of triangle)} \\ 2\beta &= 180 - \alpha \\ \beta &= \frac{180 - \alpha}{2} \end{aligned}$$

3. Construct a formula for: 4
   (a) D in terms of n, where D is the number of degrees in n right angles

   Solution: $D = 90n$

   (b) c in terms of D, where c is the number of cents in \$$D$

   Solution: $c = 100D$

   (c) m in terms of h, where m is the number of minutes in h hours

   Solution: $m = 60h$

   (d) d in terms of m, where d is the number of days in m weeks

   Solution: $d = 7m$

1. Construct a formula for: 4
   (a) the number of centimetres n in p metres

   Solution: $n = 100p$

   (b) the number of millilitres s in t litres

Solution: $s = 1000t$

(c) the number of centimetres q in $5p$ metres

Solution: $q = 500p$

(d) the number of grams x in $\frac{y}{2}$ kilograms

Solution: $x = 500y$

2. Find a formula relating x and y for each of these statements, making y th subject. 6

(a) y is three less than x.

Solution: $y = x - 3$

(b) y is four more than the square of x.

Solution: $y = x^2 + 4$

(c) y is eight times the square root of one-fifth of x.

Solution: $y = 8\sqrt{\frac{x}{5}}$

(d) x and y are supplementary angles.

Solution: $y = 180 - x$

(e) A car travelled 80 km in x hours at an average speed of y k/h.

Solution: $y = \frac{80}{x}$

(f) A car used x litres of petrol on a trip of 80 km and the fuel consumption was y litres /100 km.

Solution: $y = \frac{x}{80} \times 100$

3. Find a formula relating the given pronumerals for each of these statements. 6

(a) The number of square cm$x$ in y square metres

Solution: $x = 10000y$

(b) The selling price \$$S$ of an article with an original price of \$$m$ when a discount of 20% is given

Solution: $S = m - 0.2m$

(c) The length c cm of the hypotenuse and the lengths a cm and cm of the other two sides in a right-angled triangle

Solution: $c = \sqrt{a^2 + b^2}$

(d) The area A cm^2 of a sector of a circle with a radius of length $r =$ cm and angle θ at the centre of the circle

Solution: $A = \frac{1}{2}r^2\theta$

(e) The distance d km travelled by a car in t hours at an average speed of 75 km/h

Solution: $d = 75t$

(f) The number of hectares h in a rectangular paddock of length 400 m and width w m

Solution: $h = \frac{400w}{10000}$

5 Homework

5.1 Substitution into formulas

1. For each part, find the value of the subject when the other pronumerals have the value indicated. 3

   (a) $A = \frac{1}{2}(a+b)h$, where $a = 4, b = 6, h = 10$

   Solution: $A = \frac{1}{2}(4+6) \cdot 10 = \frac{1}{2} \cdot 10 \cdot 10 = 50$

   (b) $t = a + (n-1)d$, where $a = 30, n = 8, d = 4$

   Solution: $t = 30 + (8-1) \cdot 4 = 30 + 7 \cdot 4 = 30 + 28 = 58$

   (c) $E = \frac{1}{2}mv^2$, where $m = 8, v = 4$

   Solution: $E = \frac{1}{2} \cdot 8 \cdot 4^2 = 4 \cdot 16 = 64$

2. For each part, find the value of the subject when the other pronumerals have the value indicated. Calculate **a** – **c** correct to 3 decimal places and **d** correct to 2 4

   (a) $x = \sqrt{ab}$, where $a = 40, b = 50$

   Solution: $x = \sqrt{40 \cdot 50} = \sqrt{2000} \approx 44.721$

   (b) $V = \pi r^2 h$, where $r = 12, h = 20$

   Solution: $V = \pi \cdot 12^2 \cdot 20 = 144\pi \cdot 20 = 2880\pi \approx 9047.78$

   (c) $T = 2\pi\sqrt{\frac{\ell}{g}}$, where $\ell = 88.2, g = 9.8$

   Solution: $T = 2\pi\sqrt{\frac{88.2}{9.8}} \approx 2\pi\sqrt{9} = 2\pi \cdot 3 \approx 18.850$

   (d) $A = P(1+R)^n$, where $P = 10000, R = 0.065, n = 10$

   Solution: $A = 10000(1+0.065)^{10} \approx 10000 \cdot 1.8194 \approx 18194.00$

3. For the formula $S = 2(\ell w + \ell h + hw)$, find h if $S = 592, \ell = 10$ and $w = 8$. 1

   Solution: $592 = 2(10 \cdot 8 + 10h + 8h)$ $296 = 80 + 18h$ $216 = 18h$ $h = 12$

4. For the formula $s = ut + \frac{1}{2}at^2$, find a if $s = 1000, u = 20$ and $t = 5$ 1

Solution: $1000 = 20 \cdot 5 + \frac{1}{2}a \cdot 5^2$ $1000 = 100 + \frac{1}{2}a \cdot 25$ $900 = \frac{1}{2}a \cdot 25$ $36 = a$

5. For the formula $t = a + (n-1)d$, find n if $t = 58, d = 3$ and $a = 7$. $\boxed{1}$

Solution: $58 = 7 + (n-1) \cdot 3$ $51 = (n-1) \cdot 3$ $17 = n - 1$ $n = 18$

6. Given $v^2 = u^2 + 2ax$ and $v > 0$, find the value of v (correct to 1 decimal place) when: $\boxed{2}$

(a) $u = 0, a = 5$ and $x = 10$

Solution: $v = \sqrt{0^2 + 2 \cdot 5 \cdot 10} = \sqrt{100} = 10.0$

(b) $u = 2, a = 9.8$ and $x = 22$

Solution: $v = \sqrt{2^2 + 2 \cdot 9.8 \cdot 22} \approx \sqrt{4 + 431.2} \approx \sqrt{435.2} \approx 20.9$

7. Given $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, find the value of: $\boxed{2}$

(a) u when $f = 2$ and $v = 4$

Solution: $\frac{1}{2} = \frac{1}{u} + \frac{1}{4}$ $\frac{1}{2} - \frac{1}{4} = \frac{1}{u}$ $\frac{1}{4} = \frac{1}{u}$ $u = 4$

(b) u when $f = 3$ and $v = 4$

Solution: $\frac{1}{3} = \frac{1}{u} + \frac{1}{4}$ $\frac{1}{3} - \frac{1}{4} = \frac{1}{u}$ $\frac{1}{12} = \frac{1}{u}$ $u = 12$

8. The formula for finding the number of degrees Fahrenheit (F) for a temperature given a number of degrees Celsius (C) is $F = \frac{9}{5}C + 32$. $\boxed{6}$

Fahrenheit temperatures are still used in the USA, but in Australia we commonly use Celsius.

Calculate the Fahrenheit temperatures which people in the USA would recognise for:

(a) the freezing point of water, 0°C

Solution: $F = \frac{9}{5} \cdot 0 + 32 = 32$

(b) the boiling point of water, 100°C

Solution: $F = \frac{9}{5} \cdot 100 + 32 = 180 + 32 = 212$

(c) a nice summer temperature of 25°C

Solution: $F = \frac{9}{5} \cdot 25 + 32 = 45 + 32 = 77$

Now calculate the Celsius temperatures which people in Australia would recognise for:

(d) 50°F

Solution: $C = \frac{5}{9}(50 - 32) = \frac{5}{9} \cdot 18 \approx 10$

(e) 104°F

Solution: $C = \frac{5}{9}(104 - 32) = \frac{5}{9} \cdot 72 \approx 40$

9. Sam throws a stone down to the ground from the top of a cliff s metres high, with an initial speed of u m/s. It accelerates at a m/s^2. The stone hits the ground with a speed of v m/s given by the formula $v^2 = u^2 + 2s$. Find the speed at which the stone hits the ground, correct to 2 decimal places, if: 4

(a) $u = 0, a = 9.8$ and $s = 50$

Solution: $v = \sqrt{0^2 + 2 \cdot 9.8 \cdot 50} = \sqrt{980} \approx 31.30$

(b) $u = 5, a = 9.8$ and $s = 35$

Solution: $v = \sqrt{5^2 + 2 \cdot 9.8 \cdot 35} \approx \sqrt{25 + 686} \approx \sqrt{711} \approx 26.67$

5.2 Changing the subject of a formula

1. Given the formula $v = u + at$:

(a) rearrange the formula to make u the subject 1

Solution: $u = v - at$

(b) find the value of u when: 2

Solution:

i. $v = 20, a = 2$ and $t = 5$

Solution: $u = 20 - 2 \cdot 5 = 20 - 10 = 10$

ii. $v = 40, a = -6$ and $t = 4$

Solution: $u = 40 - (-6) \cdot 4 = 40 + 24 = 64$

(c) rearrange the formula to make a the subject 1

Solution: $a = \frac{v-u}{t}$

(d) find the value of a when: 2

Solution:

i. $v = 20, u = 15$ and $t = 2$

Solution: $a = \frac{20-15}{2} = \frac{5}{2} = 2.5$

ii. $v = -26.8, u = -14.4$ and $t = 2$

Solution: $a = \frac{-26.8-(-14.4)}{2} = \frac{-26.8+14.4}{2} = \frac{-12.4}{2} = -6.2$

iii. $v = \frac{1}{2}, u = \frac{2}{3}$ and $t = \frac{5}{6}$

Solution: $a = \frac{\frac{1}{2}-\frac{2}{3}}{\frac{5}{6}} = \frac{\frac{3}{6}-\frac{4}{6}}{\frac{5}{6}} = \frac{-\frac{1}{6}}{\frac{5}{6}} = -\frac{1}{5}$

(e) rearrange the formula to make t the subject and find t when $v = 6, u = 7$ and $a = -3$. $\boxed{2}$

Solution: $t = \frac{v-u}{a}$ $t = \frac{6-7}{-3} = \frac{-1}{-3} = \frac{1}{3}$

2. Rearrange each of these formulas to make the pronumeral in brackets the subject. $\boxed{6}$

(a) $y = mx + c$ (x)

Solution: $x = \frac{y-c}{m}$

(b) $C = 2\pi r$ (r)

Solution: $r = \frac{C}{2\pi}$

(c) $s = ut + \frac{1}{2}at^2$ (a)

Solution: $a = \frac{2(s-ut)}{t^2}$

(d) $V = \frac{1}{3}\pi r^2 h$ (h)

Solution: $h = \frac{3V}{\pi r^2}$

(e) $S = \frac{n}{2}(a + \ell)$ (n)

Solution: $n = \frac{2S}{a+\ell}$

(f) $E = mgh + \frac{1}{2}mv^2$ (h)

Solution: $h = \frac{E - \frac{1}{2}mv^2}{mg}$

3. The kinetic energy E joules of a moving object is given by $E = \frac{1}{2}mv^2$, where m kg is the mass of the object and v m/s is its speed. 4

   Rearrange the formula to make m the subject and use this to find the mass of the object when its energy and speed are, respectively:

   (a) 400 joules, 10 m/s

   Solution: $m = \frac{2E}{v^2}$ $m = \frac{2 \cdot 400}{10^2} = \frac{800}{100} = 8$

   (b) 28 joules, 4 m/s

   Solution: $m = \frac{2 \cdot 28}{4^2} = \frac{56}{16} = 3.5$

5.3 Constructing Formulas

1. Find a formula for: 8

   (a) the number of cents z in x dollars and y cents

   Solution: $z = 100x + y$

   (b) the number of minutes x in y minutes and z seconds

   Solution: $x = y + \frac{z}{60}$

   (c) the number of hours x in y minutes and z seconds

   Solution: $x = \frac{y}{60} + \frac{z}{3600}$

   (d) the cost \$$m$ of 1 book if 20 books cost \$$c$

   Solution: $m = \frac{c}{20}$

   (e) the cost \$$n$ of 1 suit if 5 suits cost \$$m$

   Solution: $n = \frac{m}{5}$

   (f) the cost \$$m$ of 1 tyre if x tyres cost \$$y$

   Solution: $m = \frac{y}{x}$

(g) the cost \$$p$ of n suits if 4 suits cost \$$k$

> **Solution:** $p = \frac{k}{4}n$

(h) the cost \$$q$ of x cars if 8 cars cost \$$b$

> **Solution:** $q = \frac{b}{8}x$

2. In each part, find a formula from the information given. [7]

(a) A hire car firm charges \$20 per day plus 40 cents per km. What is the total cost \$$C$ for a day in which x km was travelled

> **Solution:** $C = 20 + 0.40x$

(b) If there are 50 litres of petrol in the tank of a car and petrol is used at the rate of 4 litres per day, what is the number of litres y that remains after x days?

> **Solution:** $y = 50 - 4x$

(c) Cooking instructions for a forequarter of lamb are as follows: preheat oven to 220°C and cook for 45 min per kg plus an additional 20 min. What is the formula relating the cooking time T minutes and weight w kg ?

> **Solution:** $T = 45w + 20$

(d) In a sequence of numbers the first number is 2 , the second number is 4 , the third is 8 , the fourth is 16, etc. Assuming the doubling pattern continues, what the formula you would use to calculate t, the nth number?

> **Solution:** $t = 2^n$

(e) A piece of wire of length x cm is bent into a circle of area A cm^2. What is the formula relating A and x ?

> **Solution:** $A = \frac{\pi}{4\pi^2}x^2 = \frac{x^2}{4\pi}$

3. A cyclic quadrilateral has all its vertices on a circle. Its area A is given by Brahmagupta's formula [10]

$$A^2 = (s-a)(s-b)(s-c)(s-d)$$

where a, b, c and d are the side lengths of the quadrilateral and $s = \frac{a+b+c+d}{2}$ is the 'semi-perimeter'. Find the exact area of a cyclic quadrilateral with side lengths:

(a) $4, 5, 6, 7$

Solution: $s = \frac{4+5+6+7}{2} = 11$
$A^2 = (11-4)(11-5)(11-6)(11-7) = 7 \cdot 6 \cdot 5 \cdot 4$
$A = \sqrt{7 \cdot 6 \cdot 5 \cdot 4} = \sqrt{840} = 2\sqrt{210}$

(b) $7, 4, 4, 3$

Solution: $s = \frac{7+4+4+3}{2} = 9$
$A^2 = (9-7)(9-4)(9-4)(9-3) = 2 \cdot 5 \cdot 5 \cdot 6$
$A = \sqrt{2 \cdot 5 \cdot 5 \cdot 6} = \sqrt{300} = 10\sqrt{3}$

(c) $8, 9, 10, 13$

Solution: $s = \frac{8+9+10+13}{2} = 20$
$A^2 = (20-8)(20-9)(20-10)(20-13) = 12 \cdot 11 \cdot 10 \cdot 7$
$A = \sqrt{12 \cdot 11 \cdot 10 \cdot 7} = \sqrt{9240} = 2\sqrt{2310}$

(d) $39, 52, 25, 60$

Solution: $s = \frac{39+52+25+60}{2} = 88$
$A^2 = (88-39)(88-52)(88-25)(88-60) = 49 \cdot 36 \cdot 63 \cdot 28$
$A = \sqrt{49 \cdot 36 \cdot 63 \cdot 28} = \sqrt{3895584} = 1974$

(e) $51, 40, 68, 75$

Solution: $s = \frac{51+40+68+75}{2} = 117$
$A^2 = (117-51)(117-40)(117-68)(117-75) = 66 \cdot 77 \cdot 49 \cdot 42$
$A = \sqrt{66 \cdot 77 \cdot 49 \cdot 42} = \sqrt{11388348} = 3372$

6 Marking

Marker's use only.

SECTION	1	2	3	4	HW	**Total**
MARKS	$\overline{}$ 5	$\overline{}$ 42	$\overline{}$ 31	$\overline{}$ 23	$\overline{}$ 67	$\overline{}$ 168