
YEAR 9 MATHEMATICS

TOPIC 7A

: INDEX LAWS

PEN Education

2024

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1 Introduction

index	indices	power	root	exponent	reciprocal	sig fig	base	product
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1. What is the fourth power of two equal to?

1

.....

.....

.....

2. How do you interpret this?

4

.....

.....

.....

3. But then what about 2^{-2} ? Does this have an answer? What is it's interpretation?!

4

4. Why are learning about these powers?

1

2 The Index Laws

Let us begin simply with the following:

2.1 Examples:

1. Express as a power or as a product of powers.

1

(a) $5 \times 5 \times 5$

(b) $3 \times 3 \times 7 \times 7 \times 7 \times 7$

(c)

2. Express each number as a power of a prime.

1

(a) 81

(b) 128

Now some definitions:



Definition 1

Base:

.....2.....



Definition 2

Index:

.....



Laws/Results

Index Laws

Product of Powers

Now with these laws in mind, try the following 10 examples:

2.2 Examples

1. Simplify, expressing the answer in index form.

10

(a) $3^2 \times 3^4$

(f) $\frac{9^5}{9^4}$

.....

(b) $a^3 \times a^5$

(g) $10^6 \div 10^4$

.....

(c) $3x^2 \times x^3$

(h) $a^7 \div a^4$

.....

(d) $2a^2b^3 \times 5ab^2$

(i) $\frac{3y^4}{y}$

.....

(e) $\frac{3^5}{3^2}$

(j) $\frac{6x^5}{2x^3}$

.....

2. Double marks for each of these:

4

(a)

$$\frac{3x^3y^2}{4xy} \times \frac{6x^2y^3}{x^3y^2}$$

.....

(b)

$$\frac{8a^2b^3}{3a^3b} \div \frac{4ab^2}{9a^3b^5}$$

.....

.....

.....

3. And a $1.5\times$ multiplier for these:

(a) $\left(\frac{2}{3}\right)^2$

.....

.....

.....

(b) $\left(\frac{m}{n}\right)^5$

.....

.....

.....

(c) $\left(\frac{x^3}{y^2}\right)^2 \times \left(\frac{y}{x}\right)^4$

.....

.....

.....

(d) $\left(\frac{2x^2}{3}\right)^2 \div \frac{4x^3}{9}$

.....

.....

.....

(a) $(5a^3)^0$

.....

.....

.....

(b) $\frac{6x^2y}{xy^2} \times \frac{y^3x}{2y^2x^2}$

.....

.....

.....

(c) $(mn^2)^0$

(d) $(a^4b^2)^3$

(e) $(2a^4)^3$

(f) $2(x^2y)^0 \times (x^2y^3)^3$

(g)

Note: There are different possible interpretations of the word 'simplify'. There may be more than one acceptable simplified form.

2.3 Exercises:

1. State the base and index of:

(a) $6^4 =$ _____ (b) $7^3 =$ _____ (c) $8^2 =$ _____

2. Express as a power of a prime number.

(a) $8 =$ _____ (b) $27 =$ _____ (c) $64 =$ _____

3. Evaluate:

(a) $3^4 =$ _____ (b) $2^7 =$ _____ (c) $5^5 =$ _____

4. Express as a product of powers of prime numbers.

(a) $18 = \underline{\hspace{2cm}}$ (b) $24 = \underline{\hspace{2cm}}$ (c) $144 = \underline{\hspace{2cm}}$

5. Simplify:

(a) $2^7 \times 2^3$ (b) $3^3 \times 3^4 \times 3^5$ (c) $3x^2 \times 4x^3$

6. Simplify:

(a) $a^2b^3 \times b^2$ (c) $2xy^2 \times 3x^2y$
 (b) $a^3b \times a^2b^3$ (d) $4a^3b^2 \times a^2b^4$

7.

(a) $\frac{3^7}{3^2}$ (d) $\frac{10^{12}}{10^4}$
 (b) $\frac{2^6}{2^2}$ (e) $\frac{2x^3}{x^2}$
 (c) $10^7 \div 10^2$ (f) $\frac{6x^5}{2x^2}$

8.

(a) $\frac{a^3b^2}{a^2}$ (c) $\frac{12a^6b^2}{4a^2b}$
 (b) $\frac{x^3y^2}{xy}$ (d) $\frac{15xy^3}{3y^2}$

9.

(a) $\frac{a^3b^2}{ab} \times \frac{a^2b}{a}$ (c) $\frac{6ab^2}{5a^3b} \div \frac{12ab}{15a^5b}$
 (b) $\frac{x^3y}{xy^2} \times \frac{x^4y^5}{x^2}$ (d) $\frac{7x^3y^4}{2xy^2} \div \frac{21x^2y^3}{4x^3y^2}$

10.

(a) $a^4 \times \underline{\hspace{2cm}} = a^{10}$ (c) $15d^7 \div \underline{\hspace{2cm}} = 3d^2$ (e) $\ell^6m^7 \div \underline{\hspace{2cm}} = \ell^2m^5$
 (b) $9d^5 \times \underline{\hspace{2cm}} = 27d^6$ (d) $\frac{8ab^4}{24a^2b^6} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ (f) $b^7 \times \underline{\hspace{2cm}} = b^{16}$

11.

(a) a^0 (b) $2x^0$

12.

(a) $3a^0$

(c) $4a^0 + 3b^0$

(e) $(4b)^0 + 2b^0$

(b) $6a^0$

(d) $6a^0 + 7m^0$

(f) $(3b)^0 - 5d^0$

13. Simplify, leaving the answer as a power.

(a) $(2^3)^4$

(b) $(3^2)^3$

14.

(a) $(a^3)^2 \times (a^3)^4$

(c) $2ab^2 \times 3a(b^3)^2$

(b) $(x^4)^2 \times (x^3)^3$

(d) $\frac{3ab}{(b^2)^3} \times \frac{4b^7}{3a}$

15.

(a) $(3a)^2$

(c) $\left(\frac{a}{5}\right)^2$

(b) $(2x)^3$

(d) $\left(\frac{2}{x}\right)^3$

16.

(a) $(2a^2b)^2 \times 3ab^3$

(c) $(5xy^2)^3 \times (x^2y^3)^2$

(b) $(3xy^2)^3 \times (x^2y)^2$

(d) $(2a^3b)^3 \times 3a^0$

17.

(a) $\left(\frac{x^2}{y}\right)^2 \times \left(\frac{y^2}{x}\right)^3$

(c) $\frac{(3xy^2)^2 \times (2x^2y)^3}{(6x^2y)^2}$

(b) $\left(\frac{4a^2}{b}\right)^2 \times \left(\frac{b}{2a}\right)^3$

(d) $\frac{3a^2b^4 \times (2ab^2)^3}{(4a^2b^3)^2}$

3 Negative Indices

This section is tricky as negative numbers always tend to be. The most important thing you will learn here is:

$$a^{-n} = \frac{1}{a^n}$$

I want you to take a second to realise that this **breaks** your interpretation of a being raised to

some power n as being n successive multiplications of the number a .

$$a^n = \underbrace{a \times a \times a \dots}_n$$

Suddenly what does it mean to multiply a by itself $-n$ times?!
Here is some working out space for you to reconcile with this fact.

3.1 Examples:

Worked Example:

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

1. Evaluate

(a) 6^{-2}

.....

.....

.....

(c) 2^{-7}

(b) 4^{-3}

.....

.....

.....

(d) 10^{-3}

2. These ones just reciprocate!

4

3

(a) $\left(\frac{1}{3}\right)^{-1}$

(b) $\left(\frac{2}{7}\right)^{-2}$

(c) $\left(4\frac{1}{4}\right)^{-2}$

.....

.....

.....

.....

3. Write as a single power and then evaluate:

(a) $3^4 \times 3^{-2}$

.....

.....

.....

(b) $5^7 \times 5^{-8}$

.....

.....

.....

(c) $13^{-8} \times 13^{15} \times 13^{-7}$

.....

.....

.....

(d) $\left(\frac{2}{3}\right)^{-6} \times \left(\frac{2}{3}\right)^4$

.....

.....

.....

(e)

$$\frac{2^4}{2^5}$$

(g)

$$\frac{5}{5^3}$$

.....

.....

.....

.....

.....

.....

(f)

$$\frac{3^4}{3^7}$$

(h)

$$\frac{3^4}{3^6}$$

.....

.....

.....

.....

.....

.....

4. Simplify, expressing the answers with positive indices

(a) $a^2b^{-3} \times a^{-4}b^5$

(c) $(2a^{-2}b^3)^{-2}$

.....

.....

.....

.....

(b) $\frac{x^2y^3}{x^3y^2}$

(d) $\left(\frac{3m^2}{n}\right)^{-4}$

.....

.....

.....

.....

4

Note that in general $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$.

3.2 Exercises:

1. Express with a positive index and then evaluate.

4

(a) 2^{-1}

(c) 2^{-4}

.....
.....
.....

(b) 5^{-1}

(d) 3^{-3}

.....
.....
.....

2. Write each fraction as a power of a prime with a negative index.

4

(a) $\frac{1}{8}$

(c) $\frac{1}{16}$

.....
.....
.....

(b) $\frac{1}{9}$

(d) $\frac{1}{64}$

.....
.....
.....

3. Express with positive indices, evaluating where possible.

4

(a) a^{-3}

(c) $\frac{3}{a^{-4}}$

.....
.....
.....

(b) x^{-7}

(d) $\frac{5}{x^{-5}}$

.....
.....
.....

4. Simplify and then evaluate

(a) $\left(\frac{1}{4}\right)^{-1}$

.....

(c) $3^5 \times 3^{-2}$

.....

(b) $\left(\frac{2}{5}\right)^{-2}$

.....

(d) $5^{11} \times 5^{-8}$

.....

5.

(a) $\frac{2^3}{2^6}$

.....

(c) $\frac{8^6}{8^7}$

.....

(b) $\frac{4^2}{4^4}$

.....

(d) $\frac{20^4}{20^6}$

.....

6. Express with negative index.

(a) $\frac{3}{x}$

.....

(b) $\frac{5}{x^2}$

.....

(c) $\frac{8}{x^4}$

.....

7. Evaluate.

(a) $\left(\frac{1}{2}\right)^{-1}$

.....

(b) $\left(\frac{2}{3}\right)^{-1}$

.....

(c) $\left(\frac{1}{2}\right)^{-2}$

.....

8. Simplify, expressing the answer with positive indices.

(a) $x^{-6}y^4 \times x^2y^{-2}$

(c) $\frac{8a^{-4}}{2a^6}$

.....	
.....	
.....	

(b) $2a^{-1}b^5 \times 7ab^{-3}$

(d) $\frac{36h^{-9}}{9h^{-4}}$

.....	
.....	
.....	

9. Fill in the missing term

(a) $6^4 \times \dots = 6^2$

(d) $(a^5) \dots = a^{-15}$

.....	
.....	
.....	

(b) $m^5 \times \dots = m^{-6}$

(e) $(\dots)^{-2} = \frac{m^6}{25}$

.....	
.....	
.....	

(c) $d^{-7} \div \dots = d^{15}$

(f) $(\dots)^{-2} = p^4q^{-6}$

.....	
.....	
.....	

10. Simplify, expressing the answers with positive indices. Evaluate powers where possible.

$$(a) \quad (3a^2b^{-2})^3 \times (2a^4)^{-2}$$

.....

$$(c) \quad \frac{(2a^4b^{-2})^3}{c^2} \times \frac{(2^2a^{-3}b^2)^{-1}}{c}$$

.....

$$(b) \quad (6a^5b^{-4})^{-3} \times 2(a^3b^{-3})^2$$

.....

$$(d) \quad \mathbf{j} \frac{(2a^4)^2}{b^7} \div \frac{(a^2)^{-3}}{2b}$$

.....

4 Homework

The Index Laws

1. State the base and index of:

3

(a) 10^4

(b) 5

(c) 6^0

.....
.....
.....

2. Express as a power of a prime number.

3

(a) 243

(b) 125

(c) 81

.....
.....
.....

3. Evaluate:

3

(a) 7^4

(b) $2^3 \times 3^5$

(c) $6^4 \times 3^2$

.....
.....
.....

4. Express as a product of powers of prime numbers.

3

(a) 90

(b) 700

(c) 84

.....
.....
.....

5. Simplify, leaving the answer as a power or a product of powers.

6

(a) $3^4 \times 3^5$

(c) $a^3 \times a^8$

(e) $2y \times 3y^4$

.....
.....
.....

(b) $3^4 \times 3^7$

(d) $b^7 \times b^{12}$

(f) $4b^2 \times 3b^4$

.....
.....
.....

6. Simplify:

2

(a) $x^2y \times x^3y$

(b) $5a^4b \times 2ab^3$

.....

7. Simplify, leaving the answer as a power or a product of powers.

6

(a) $5^4 \div 5$

(d) $\frac{a^5}{a^3}$

.....

(b) $7^5 \div 7^3$

(e) $\frac{10y^{12}}{5y^3}$

.....

(c) $\frac{a^4}{a}$

(f) $\frac{27p^4}{9p}$

.....

8. Simplify:

4

(a)

$$\frac{a^5b^3}{a^4b}$$

(c)

$$\frac{16a^4b^3}{12a^2b^2}$$

.....

(b)

$$\frac{x^4y^7}{x^3y^2}$$

(d)

$$\frac{27x^2y^3}{18xy^2}$$

.....

9. Simplify:

4

(a)

$$\frac{2ab^2}{3a^2b^4} \times \frac{6a^4b^5}{ab}$$

(c)

$$\frac{14a^4b^3}{3ab^2} \div \frac{7a^5b^4}{6a^3b^5}$$

.....

.....

.....

(b)

$$\frac{12x^4y^3}{3x^2y} \times \frac{x^2y^4}{x^3y^5}$$

(d)

$$\frac{12x^2y}{x^3y^4} \div \frac{6xy^2}{x^6y^7}$$

.....

.....

.....

10. Copy and complete.

9

(a) $4a^3 \times \underline{\hspace{2cm}} = 12a^7$

(d) $9d^6 \div \underline{\hspace{2cm}} = 3d$

(g) $14x^5y^2 \times \underline{\hspace{2cm}} = 42x^{10}y^5$

.....

.....

.....

(b) $a^8 \div \underline{\hspace{2cm}} = a^4$

(e) $m^4n^5 \times \underline{\hspace{2cm}} = m^{10}n^7$

(h) $9m^7n^4 \div \underline{\hspace{2cm}} = 3m^2$

.....

.....

.....

(c) $x^{10} \div \underline{\hspace{2cm}} = x^6$

(f) $a^7b^4 \div \underline{\hspace{2cm}} = a^2b$

(i) $18p^2q^6 \div \underline{\hspace{2cm}} = 3pq$

.....

.....

.....

11. Simplify:

2

(a) xy^0

(b) $7x^0y^0$

.....

.....

.....

12. Simplify:

6

(a) $(4a)^0$

(d) $(4a + 3b)^0$

.....

(b) $(3b)^0$

(e) $(5m^0 + 7b)^0$

.....

(c) $(2a + 1)^0$

(f) $(6m - 2c^0)^0$

.....

13. Simplify, leaving the answer as a power.

2

(a) $(a^2)^5$

(b) $(y^5)^6$

.....

14. Simplify

1

$$\frac{(y^3)^4}{(y^4)^2}$$

.....

15. Simplify

1

$$\frac{3(x^3y)^2}{(x^2y)^2} \div \frac{12x^4y^2}{(2x^3y)^2}$$

.....

16. Copy and complete (using index law 3).

6

(a) $(a^6)\text{---} = a^{24}$

.....

(b) $(b^3)\text{-----} = b^{21}$

(e) $(\text{-----})^4 = a^8$

.....

(c) $(m^6)\text{-----} = m^{30}$

(f) $(\text{-----})^3 = m^{15}$

.....

(d) $(\text{-----})^6 = p^{36}$

.....

17.

6

(a) Is it true that $(a^2)^6 = (a^6)^2$?

.....

(b) Is it true that $(b^4)^7 = (b^7)^4$?

.....

(c) Generalise your result.

.....

18. Simplify by expanding the brackets.

4

(a) $(xy^3)^2$

(c) $\left(\frac{a}{b}\right)^5$

.....

(b) $(a^2b)^4$

(d) $\left(\frac{x^2}{y}\right)^3$

.....

19. Simplify:

(a)

$$\left(\frac{x^3}{y^2}\right)^2 \div \left(\frac{x}{y^2}\right)^3$$

.....

(b)

$$\left(\frac{2x^4}{y}\right)^5 \div \left(\frac{4x^3}{y^3}\right)^2$$

.....

20. Simplify:

(a)

$$\frac{(2x^2y^3)^3 \times (5xy^2)^2}{(10x^2y)^2 \times (xy)^3}$$

.....

(b)

$$\frac{(6ab)^3 \times 2a^7b^4}{(2ab)^4 \times (3a^2b)^2}$$

.....

21. Copy and complete.

(a) $(a^2b^3)^4 = a^8b^{12}$

(d) $(x^4y^7)^{-1} = 1$

(g) $(7m^3)^2 = 49m^6$

(b) $(m^5n^4)^6 = m^{30}n^{24}$

(e) $(2a^2)^4 = 16a^8$

(h) $(4\ell^3m)^3 = 64\ell^9m^3$

(c) $(p^3q)^3 = p^9q^3$

(f) $(3q^3)^3 = 27q^9$

(i) $(5m^5n^3)^2 = 25m^{10}n^6$

Negative Indices

1. Express with a positive index and then evaluate.

(a) 3^{-2}

.....
.....
.....

(d) 5^{-3}

.....
.....
.....

(b) 6^{-2}

.....
.....
.....

(e) 3^{-4}

.....
.....
.....

(c) 9^{-2}

.....
.....
.....

(f) 10^{-5}

.....
.....
.....

2. Write each fraction as a power of a prime with a negative index.

6

(a) $\frac{1}{27}$

.....
.....
.....

(d) $\frac{1}{125}$

.....
.....
.....

(b) $\frac{1}{49}$

.....
.....
.....

(e) $\frac{1}{169}$

.....
.....
.....

(c) $\frac{1}{121}$

.....
.....
.....

(f) $\frac{1}{81}$

.....
.....
.....

3. Express with positive indices, evaluating where possible.

6

(a) $3a^{-4}$

.....
.....
.....

.....
.....
.....

(b) $5x^{-7}$

(c) $4a^{-5}$

(e) $3^{-2}a^{-2}$

.....

(d) $\frac{1}{x^{-3}}$

(f) $4^{-2}x^{-2}$

.....

4. Simplify where possible and then evaluate.

4

(a) $(3\frac{1}{3})^{-2}$

(c) $7^3 \times 7^{-5}$

.....

(b) $(\frac{2}{3})^{-3}$

(d) $4^3 \times 4^{-5}$

.....

5. Write as a single power and then evaluate.

6

(a)

$$\frac{3^8}{3^9}$$

.....

.....(d)

.....

$$\frac{5^7}{5^{10}}$$

(b)

$$\frac{6^5}{6^8}$$

.....

.....(e)

.....

$$\frac{3^5}{3^9}$$

(c)

$$\frac{7^1}{7^3}$$

.....

(f)

$$\frac{12^{12}}{12^{14}}$$

.....
.....
.....

6. Express with negative index.

3

(a) $\frac{3}{2x^4}$

(b) $\frac{4}{3x^7}$

(c) $\frac{2}{3x^5}$

.....
.....
.....

7. Evaluate.

7

(a) $\left(\frac{4}{5}\right)^{-2}$

(b) $\left(2\frac{1}{4}\right)^{-2}$

(c) $\left(1\frac{1}{5}\right)^{-3}$

.....
.....
.....

8. Simplify, expressing the answer with positive indices.

9

(a) $a^{-3}b^{-5} \times a^5b^{-3}$

(d) $3r^2s^3 \times 4r^{-3}s^{-5}$

(g) $\frac{27m^{-3}}{9m^{-2}}$

.....
.....
.....

(b) $3x^{-2}y^5 \times 5x^{-7}y^{-2}$

(e) $\frac{16a^{-4}}{8a^5}$

(h) $\frac{72a^4b^{-3}}{36ab^{-2}}$

.....
.....
.....

(c) $7a^3m^{-4} \times 8a^{-5}m^{-3}$

(f) $\frac{18a^{-4}}{4a^5}$

(i) $\frac{7a^2b^{-3}c^{-4}}{21a^5b^{-7}c^{-9}}$

.....
.....
.....

9. Copy and complete.

9

(a) $9^5 \times 9^{-1} = 9^4$

(d) $b^7 \div b^{-8} = b^{15}$

(g) $(3a^3)^{-3} = \frac{1}{27a^9}$

(b) $b^9 \times b^{-2} = b^7$

(e) $e^{-7} \div e^2 = e^{-5}$

(h) $\left(\frac{a^2}{b^3}\right)^{-3} = \frac{a^6}{b^9}$

(c) $a^{11} \div a^{-3} = a^{14}$

(f) $(m^{-2})^{-5} = m^{10}$

(i) $\left(\frac{m^2n^3}{p}\right)^{-6} = \frac{m^{12}n^{18}}{p^6}$

10. Simplify, expressing the answers with positive indices. Evaluate powers where possible.

(a)

$$(5x^4y^6)^{-3} \times (5^2xy^{-1})^3$$

.....

.....

.....

(e)

$$\frac{(m^2n^3)^2}{p^{-3}} \times (mnp^{-2})^{-3}$$

.....

.....

.....

(b)

$$(5m^2n^{-3})^{-2} \times 2(m^{-2}n^3)^2$$

.....

.....

.....

.....

.....

.....

..... (f)

.....

.....

(g)

$$\frac{(x^2)^2}{y} \times \frac{(y^2)^{-3}}{x^3}$$

.....

.....

.....

(c)

$$\frac{(2x^3)^{-2}}{y^4} \times \frac{(2x^7)^2}{3y^5}$$

.....

.....

.....

.....

.....

.....

(d)

$$\frac{(3m^2n^3)^{-2}}{p^4} \div \frac{p^{-3}}{m}$$

.....

.....

.....

.....

.....

.....

5 Marking

Marker’s use only.

SECTION	1	2	3	HW	Total
MARKS	$\overline{10}$	$\overline{87}$	$\overline{59}$	$\overline{146}$	$\overline{302}$