Year 9 Mathematics
Topic Test 4
Formulae & Indices

PEN Education

2024

Contents

1 Introduction

Today we are going to spend an hour doing an in-class topic test. It is well known how much you all *despise* such **tests**, but according to the literature, testing yourself is the most effective way to learn!

You should acknowledge that you have now just completed all of the theory surrounding formula rearrangements, substitutions within such formulae as well as indice arithmetic, index laws and scientific notation / significant figures. And if you have been attempting your homework you should be able to independently answer questions about these topics in a quiet and timed environment.

Before we begin, here are the **buzz words** from the assessed topics, if you do not understand any word here ASK NOW!

formula subject pronumeral substitution expression construction equation rearrange

index indices power root exponent reciprocal sig fig base product

2 Problems

2.1 Formulae

1. Temperatures can be measured in either degrees Fahrenheit or degrees Celsius. To convert from one scale t the other, the following formula is used: $F = \frac{9}{5}C + 32$. 3

   (a) Rearrange the formula to make C the subject.

   Solution: $C = \frac{5}{9}(F - 32)$

   (b) On a particular day in Melbourne, the temperature was 28°C. What is this temperature measured in Fahrenheit?

   Solution: $F = \frac{9}{5} \times 28 + 32 = 82.4°\text{F}$

   (c) In Boston, USA, the minimum overnight temperature was 4°F. What is this temperature measured in Celsius?

   Solution: $C = \frac{5}{9}(4 - 32) = -15.56°\text{C}$

   (d) What number represents the same temperature in °C and °F ?

   Solution: $C = F$
   $\frac{5}{9}(F - 32) = F$
   $5F - 160 = 9F$
   $4F = 160$
   $F = 40°$

   (e) An approximate conversion formula, used frequently when converting oven temperatures, is $F = 2C + 30$. Use this to convert these temperatures:

      i. an oven temperature of 180°C

      Solution: $F = 2 \times 180 + 30 = 390°\text{F}$

      ii. an oven temperature of 530°F

      Solution: $C = \frac{530-30}{2} = 250°\text{C}$

2. Gareth the gardener has a large rectangular vegetable patch and he wishes to put in a path around it usin concrete pavers that measure 50 cm × 50 cm. The path is to be 1 paver wide. Let n be the numb of pavers required. If the vegetable patch measures x metres by y metres, find a formula for n in terms of x a y. [2]

Solution: $n = 2(x + y) \times 2$ because each side needs $x \times 2$ and $y \times 2$ pavers, and there are two lengths and two widths.

3. If $s = \frac{n}{2}(2a + (n-1)d)$: [3]

   (a) find the value of s when $n = 10, a = 6$ and $d = 3$

   Solution: $s = \frac{10}{2}(2 \times 6 + (10-1) \times 3) = 5(12 + 27) = 5 \times 39 = 195$

   (b) find the value of a when $s = 350, n = 20$ and $d = 4$

   Solution: $350 = \frac{20}{2}(2a + (20-1) \times 4)$
   $350 = 10(2a + 76)$
   $35 = 2a + 76$
   $a = \frac{35-76}{2} = -20.5$

   (c) find the value of d when $s = 460, n = 10$ and $a = 10$

   Solution: $460 = \frac{10}{2}(2 \times 10 + (10-1)d)$
   $460 = 5(20 + 9d)$
   $92 = 20 + 9d$
   $d = \frac{92-20}{9} = 8$

4. The formula for the geometric mean m of two positive numbers a and b is $m = \sqrt{ab}$. [2]

   (a) Find m if $a = 16$ and $b = 25$.

   Solution: $m = \sqrt{16 \times 25} = \sqrt{400} = 20$

   (b) Find a if $m = 7$ and $b = 16$.

   Solution: $7 = \sqrt{a \times 16}$
   $49 = a \times 16$
   $a = \frac{49}{16} = 3.0625$

5. If $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$: [2]

   (a) find x if $b = 4, a = 1$ and $c = -24$

Solution: $x = \frac{-4+\sqrt{4^2-4\times1\times(-24)}}{2\times1}$
$x = \frac{-4+\sqrt{16+96}}{2}$
$x = \frac{-4+\sqrt{112}}{2}$
$x = \frac{-4+10.58}{2}$
$x = 3.29$

(b) find c if $a = 1, x = 6$ and $b = 2$

Solution: $6 = \frac{-2+\sqrt{2^2-4\times1\times c}}{2\times1}$
$12 = -2 + \sqrt{4-4c}$
$14 = \sqrt{4-4c}$
$196 = 4 - 4c$
$c = \frac{4-196}{4} = -48$

6. A pillar is in the shape of a cylinder with a hemispherical top. If r metres is the radius of the bas and h metres is the total height, the volume V cubic metres is given by the formula $V = \frac{1}{3}\pi r^2(3h - r)$ [2]

(a) Rearrange the formula to make h the subject.

Solution: $V = \frac{1}{3}\pi r^2(3h - r)$
$3V = \pi r^2(3h - r)$
$3V = 3\pi r^2 h - \pi r^3$
$3V + \pi r^3 = 3\pi r^2 h$
$h = \frac{3V+\pi r^3}{3\pi r^2}$

(b) Find the height of the pillar, correct to the nearest centimetre, if the radius of the pillar is 0.5 m and the volume is 10 m^3.

Solution: $h = \frac{3\times10+\pi\times0.5^3}{3\pi\times0.5^2}$
$h = \frac{30+\frac{1}{8}\pi}{\frac{3}{4}\pi}$
$h = \frac{30+0.3927}{2.3562}$
$h = \frac{30.3927}{2.3562}$
$h = 12.90$ m
$h \approx 1290$ cm

7. Rearrange each of these formulas to make the pronumeral in brackets the subject. (All of the pronumerals represent positive numbers.) [3]

(a) $A = \ell \times w$ (ℓ)

Solution: $\ell = \frac{A}{w}$

(b) $V = \pi r^2 h$ (r)

Solution: $r = \sqrt{\frac{V}{\pi h}}$

(c) $\frac{1}{x} + \frac{1}{y} = \frac{2}{z}$ (z)

Solution: $\frac{1}{x} + \frac{1}{y} = \frac{2}{z}$
$\frac{y+x}{xy} = \frac{2}{z}$
$z = \frac{2xy}{x+y}$

8. If a stone is dropped off a cliff, the number of metres it has fallen after a certain number of seconds i found by multiplying the square of the number of seconds by 4.9. [2]

(a) Find the formula for the distance d metres fallen by the stone in t seconds.

Solution: $d = 4.9t^2$

(b) Find the distance fallen in 1.5 seconds.

Solution: $d = 4.9 \times 1.5^2 = 4.9 \times 2.25 = 11.025$ m

9. If $t = \sqrt{\frac{M}{M-m}}$: [3]

(a) express the formula with m as the subject

Solution: $t^2 = \frac{M}{M-m}$
$t^2(M-m) = M$
$t^2M - t^2m = M$
$t^2m = t^2M - M$
$m = \frac{t^2M-M}{t^2}$

(b) express the formula with M as the subject

Solution: $t^2 = \frac{M}{M-m}$
$t^2(M-m) = M$
$M - t^2m = M/t^2$
$M(1 - 1/t^2) = t^2m$
$M = \frac{t^2m}{1-1/t^2}$

(c) find the value of M if $m = 3$ and $t = \sqrt{2}$.

Solution: $M = \frac{(\sqrt{2})^2 \times 3}{1-1/(\sqrt{2})^2}$
$M = \frac{2\times 3}{1-1/2}$
$M = \frac{6}{1/2}$
$M = 12$

10. The total surface area S cm^2 of a cylinder is given in terms of its radius r cm and height h cm by the formula $S = 2\pi r(r + h)$. [2]

    (a) Express this formula with h as the subject.

    Solution: $S = 2\pi r^2 + 2\pi rh$
    $h = \frac{S-2\pi r^2}{2\pi r}$

    (b) What is the height of such a cylinder if the radius is 7 cm and the total surface area is 50 cm^2 ? Calculate your answer in centimetres, correct to 2 decimal places.

    Solution: $h = \frac{500-2\pi\times 7^2}{2\pi\times 7}$
    $h = \frac{500-2\pi\times 49}{2\pi\times 7}$
    $h = \frac{500-307.88}{43.98}$
    $h = \frac{192.12}{43.98}$
    $h = 4.37$ cm

11. The sum S of the squares of the first n whole numbers is given by the formula $S = \frac{n(n+1)(2n+1)}{6}$. Find the sum of the squares of: [2]

    (a) the first 20 whole numbers

    Solution: $S = \frac{20(20+1)(2\times 20+1)}{6}$
    $S = \frac{20\times 21\times 41}{6}$
    $S = \frac{17220}{6}$
    $S = 2870$

    (b) all the numbers from 5 to 21 inclusive

    Solution: $S_{21} = \frac{21(21+1)(2\times 21+1)}{6}$
    $S_{21} = \frac{21\times 22\times 43}{6}$
    $S_{21} = 3311$
    $S_4 = \frac{4(4+1)(2\times 4+1)}{6}$
    $S_4 = \frac{4\times 5\times 9}{6}$
    $S_4 = 30$
    $S = S_{21} - S_4 = 3311 - 30 = 3281$

12. For the formula $D = \sqrt{\frac{f+x}{f-x}}$, make x the subject. [2]

Solution: $D^2 = \frac{f+x}{f-x}$
$D^2(f-x) = f+x$
$D^2 f - D^2 x = f + x$
$D^2 x + x = D^2 f - f$
$x(D^2+1) = D^2 f - f$
$x = \frac{D^2 f - f}{D^2+1}$

13. Cans in a supermarket are displayed in a triangular stack with one can at the top, two cans in the second row from the top, three cans in the third row from the top, and so on. What is the number of cans in the display if the number of rows is: [2]

(a) n

Solution: The number of cans is given by the nth triangular number, which is $\frac{n(n+1)}{2}$.

(b) 35

Solution: For $n = 35$, the number of cans is $\frac{35(35+1)}{2} = \frac{35\times 36}{2} = 630$.

14. Rearrange this formula to make the pronumeral in brackets the subject. [2]

$$v^2 = u^2 + 2as \quad (u)$$

Solution: To make u the subject, we rearrange the formula: $u = \sqrt{v^2 - 2as}$.

15. For the formula $I = \frac{180n-360}{n}$: [2]

(a) find I when $n = 6$

Solution: For $n = 6$, $I = \frac{180\times 6-360}{6} = \frac{1080-360}{6} = \frac{720}{6} = 120$.

(b) make n the subject of the formula and find n when $I = 108$

Solution: To make n the subject, we rearrange the formula: $n = \frac{180}{I+2}$. For $I = 108$, $n = \frac{180}{108+2} = \frac{180}{110} = \frac{18}{11}$.

16. Find the formula connecting x and y for each of these statements making y the subject. [4]

(a) y is four more than twice the square of x.

Solution: $y = 2x^2 + 4$.

(b) x and y are complementary angles.

Solution: $y = 90 - x$.

(c) A car travelled x km in y hours at a speed of 100 km/h.

Solution: $y = \frac{x}{100}$.

(d) A car travelled 100 km in y hours at a speed of x km/h.

Solution: $y = \frac{100}{x}$.

2.1.1 Challenge Problems

17. A builder wishes to place a circular cap of a given height above an existing window. To do this he needs to know the location of the centre of the circle (the cap is not necessarily a semicircle) and the radius of the circle. O is the centre of the required circle, the radius of the required circle is r cm, the width of the window is $2d$ cm and the height of the circular cap is h cm. 10

(a) Express each of these in terms of r, d and h.

i. AB

Solution: $AB = 2d$.

ii. OA

Solution: $OA = r$.

(b) Show that $r = \frac{h^2+d^2}{2h}$.

Solution: By the intersecting chords theorem, $OA^2 = OD \times OB$, where $OD = r - h$ and $OB = r + h$. So, $r^2 = (r - h)(r + h) = r^2h^2$. Adding h^2 to both sides gives $r^2 + h^2 = 2rh$. Dividing by $2h$ gives $r = \frac{h^2+d^2}{2h}$.

(c) If the window is 120 cm wide and the cap is 40 cm high, find:

i. the radius of the circle

Solution: $r = \frac{40^2+60^2}{2\times 40} = \frac{1600+3600}{80} = \frac{5200}{80} = 65$ cm.

ii. how far below the top of the window the centre of the circle must be placed

Solution: $OD = r - h = 65 - 40 = 25$ cm.

(d) If the builder used a circle of radius 50 cm and this produced a cap of height 20 cm, what was the width of the window?

Solution: $d = \sqrt{2rh - h^2} = \sqrt{2 \times 50 \times 20 - 20^2} = \sqrt{2000 - 400} = \sqrt{1600} =$ 40 cm. So the width of th window is $2d = 80$ cm.

18. A group of n people attend a club meeting. Before the meeting begins, they all shake hands with each other. Write a formula t find H, the number of handshakes exchanged. [4]

Solution: Each person shakes hands with $n-1$ others, and there are n people, so there are $n(n-1)$ handshakes. However, this counts each handshake twice, so $H = \frac{n(n-1)}{2}$.

2.2 Indices

1. Evaluate: [2]

(a) 2^6

Solution: $2^6 = 64$.

(b) 10^6

Solution: $10^6 = 1,000,000$.

2. Express 120^2 as a product of powers of prime numbers. [1]

Solution: $120^2 = (2^3 \times 3 \times 5)^2 = 2^6 \times 3^2 \times 5^2$.

3. Simplify and evaluate where possible. [8]

(a) $a^6 \times a^7$

Solution: $a^6 \times a^7 = a^{6+7} = a^{13}$.

(b) $2x^3 \times 5x^6$

Solution: $2x^3 \times 5x^6 = 10x^{3+6} = 10x^9$.

(c) $a^7 \div a^4$

Solution: $a^7 \div a^4 = a^{7-4} = a^3$.

(d) $\frac{12b^7}{6b^2}$

Solution: $\frac{12b^7}{6b^2} = 2b^{7-2} = 2b^5$.

(e) $\frac{18p^{10}}{9p}$

Solution: $\frac{18p^{10}}{9p} = 2p^{10-1} = 2p^9$.

(f) $(a^4)^3$

Solution: $(a^4)^3 = a^{4\times3} = a^{12}$.

(g) $(2a^7)^3$

Solution: $(2a^7)^3 = 2^3a^{7\times3} = 8a^{21}$.

(h) $3b^0$

Solution: $3b^0 = 3 \times 1 = 3$.

4. Simplify and evaluate where possible. 7

(a)

$$4a^2b^3 \times 5ab^4$$

Solution: $4a^2b^3 \times 5ab^4 = 20a^{2+1}b^{3+4} = 20a^3b^7$.

(b)

$$\frac{20a^4b^2}{5a^2b}$$

Solution: $\frac{20a^4b^2}{5a^2b} = 4a^{4-2}b^{2-1} = 4a^2b$.

(c)

$$\frac{24m^9n^4}{18m^6n^2}$$

Solution: $\frac{24m^9n^4}{18m^6n^2} = \frac{4}{3}m^{9-6}n^{4-2} = \frac{4}{3}m^3n^2$.

(d)

$$\left(3a^3b\right)^4$$

Solution: $(3a^3b)^4 = 3^4a^{3\times4}b^4 = 81a^{12}b^4$.

(e)

$$\left(5a^2b\right)^2 \times 4a^4b^3$$

Solution: $(5a^2b)^2 \times 4a^4b^3 = 25a^4b^2 \times 4a^4b^3 = 100a^8b^5$.

(f)

$$\frac{8m^4n^2}{7m^3n} \div \frac{3m^3n^5}{14m^9n^{16}}$$

Solution: $\frac{8m^4n^2}{7m^3n} \div \frac{3m^3n^5}{14m^9n^{16}} = \frac{8m^4n^2}{7m^3n} \times \frac{14m^9n^{16}}{3m^3n^5} = \frac{8\times14m^{10}n^{17}}{7\times3m^6n^6} = \frac{112m^4n^{11}}{21} = \frac{16}{3}m^4n^{11}$.

(g)

$$\frac{(2x^2y)^3}{5x^6y^2} \times \left(\frac{x^3}{2y^2}\right)^3$$

Solution: $\frac{(2x^2y)^3}{5x^6y^2} \times \left(\frac{x^3}{2y^2}\right)^3 = \frac{8x^6y^3}{5x^6y^2} \times \frac{x^9}{8y^6} = \frac{8}{5}y \times \frac{x^9}{8y^6} = \frac{x^9}{5y^5}$.

5. Evaluate: 6

(a) 6^{-2}

Solution: $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$.

(b) 8^{-3}

Solution: $8^{-3} = \frac{1}{8^3} = \frac{1}{512}$.

(c) 2^{-7}

Solution: $2^{-7} = \frac{1}{2^7} = \frac{1}{128}$.

(d) $\left(\frac{4}{5}\right)^{-2}$

Solution: $\left(\frac{4}{5}\right)^{-2} = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$.

(e) $\left(\frac{2}{3}\right)^{-4}$

Solution: $\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$.

(f) $\left(16^{\frac{1}{4}}\right)^{-3}$

Solution: $\left(16^{\frac{1}{4}}\right)^{-3} = (2)^{-3} = \frac{1}{2^3} = \frac{1}{8}$.

6. Simplify, expressing the answer with positive indices. 4

(a) $\frac{4m^2n^5p^{-6}}{16m^{-2}n^5p^3}$

Solution: $= \frac{4}{16} \cdot m^{2-(-2)} \cdot n^{5-5} \cdot p^{-6-3}$
$= \frac{1}{4} \cdot m^4 \cdot p^{-9}$
$= \frac{m^4}{4p^9}$

(b) $(2^2y^3)^{-5}$

Solution: $= (2^{2\cdot-5}) \cdot (y^{3\cdot-5})$
$= 2^{-10} \cdot y^{-15}$
$= \frac{1}{2^{10}y^{15}}$

(c) $(5^{-2}x^3)^{-5}$

Solution: $= (5^{-2\cdot-5}) \cdot (x^{3\cdot-5})$
$= 5^{10} \cdot x^{-15}$
$= \frac{5^{10}}{x^{15}}$

(d) $(3^{-3}a^2b^{-1})^{-4}$

Solution: $= (3^{-3\cdot-4}) \cdot (a^{2\cdot-4}) \cdot (b^{-1\cdot-4})$
$= 3^{12} \cdot a^{-8} \cdot b^4$
$= \frac{3^{12}b^4}{a^8}$

7. Simplify, expressing the answer with positive indices. 3

(a) $4a^2 \times 5a^{-3}$

Solution: $= 4 \cdot 5 \cdot a^{2-3}$
$= 20a^{-1}$
$= \frac{20}{a}$

(b) $14a^{-4} \div 7a^{-5}$

Solution: $= \frac{14}{7} \cdot a^{-4-(-5)}$
$= 2a^1$
$= 2a$

(c) $\frac{2m^3n^4}{(5m)^2} \times \frac{10m}{3n^{-4}}$

Solution: $= \frac{2m^3n^4}{25m^2} \cdot \frac{10m}{3n^{-4}}$
$= \frac{2}{25} \cdot \frac{10}{3} \cdot m^{3-2+1} \cdot n^{4+4}$
$= \frac{20}{75} \cdot m^2 \cdot n^8$
$= \frac{4}{15}m^2n^8$

8. Evaluate: 3

(a) $49^{\frac{1}{2}}$

Solution: $= \sqrt{49}$
$= 7$

(b) $125^{\frac{2}{3}}$

Solution: $= \sqrt[3]{125^2}$
$= \sqrt[3]{15625}$
$= 25$

(c) $\left(\frac{1}{8}\right)^{-\frac{2}{3}}$

Solution: $= (8^{\frac{2}{3}})$
$= \sqrt[3]{8^2}$
$= \sqrt[3]{64}$
$= 4$

9. Simplify, expressing the answer with positive indices. 4

(a) $3b^{\frac{2}{3}} \times 4b$

Solution: $= 3 \cdot 4 \cdot b^{\frac{2}{3}+1}$
$= 12b^{\frac{5}{3}}$

(b) $p^{\frac{2}{3}} \div p^{\frac{1}{2}}$

Solution: $= p^{\frac{2}{3}-\frac{1}{2}}$
$= p^{\frac{4}{6}-\frac{3}{6}}$
$= p^{\frac{1}{6}}$

(c) $\left(2x^{-\frac{1}{3}}\right)^{-2}$

Solution: $= 2^{-2} \cdot x^{\frac{2}{3}}$
$= \frac{1}{4}x^{\frac{2}{3}}$

(d) $(8p^{-2}q^3)^{\frac{1}{2}}$

Solution: $= 8^{\frac{1}{2}} \cdot p^{-1} \cdot q^{\frac{3}{2}}$
$= 2 \cdot \frac{q^{\frac{3}{2}}}{p}$

10. Write in scientific notation. 3

(a) 164000000

Solution: $= 1.64 \times 10^8$

(b) 0.0047

Solution: $= 4.7 \times 10^{-3}$

(c) 0.0035

Solution: $= 3.5 \times 10^{-3}$

11. Write in decimal form. [3]

(a) 6.8×10^4

Solution: $= 68000$

(b) 9.4×10^{-2}

Solution: $= 0.094$

(c) 3.2×10^{-4}

Solution: $= 0.00032$

12. Simplify, writing each answer in scientific notation. [2]

(a) $(3.1 \times 10^4) \times (2 \times 10^{-2})$

Solution: $= 3.1 \cdot 2 \times 10^{4-2}$
$= 6.2 \times 10^2$

(b) $\frac{(3\times10^4)^3}{9\times10^{-2}}$

Solution: $= \frac{3^3\times10^{12}}{9\times10^{-2}}$
$= \frac{27}{9} \times 10^{12+2}$
$= 3 \times 10^{14}$

13. Write in scientific notation correct to the number of significant figures indicated in the brackets. [6]

(a) 18.62 (2)

Solution: $= 1.9 \times 10^{1}$

(b) 18.62 (3)

Solution: $= 1.86 \times 10^{1}$

(c) 18.62 (1)

Solution: $= 2 \times 10^{1}$

(d) 0.004276 (2)

Solution: $= 4.3 \times 10^{-3}$

(e) 5973.4 (2)

Solution: $= 6.0 \times 10^{3}$

(f) 0.473952 (3)

Solution: $= 4.74 \times 10^{-1}$

3 Homework

This week homework is a little different. The only thing that will be marked from you is a reattempt of every single question that you got incorrect on the class quiz. Tutors should have handed out extra lined paper.

4 Marking

Marker's use only.

SECTION	1	2	HW	**Total**
MARKS	$\frac{}{0}$	$\frac{}{104}$	$\frac{}{0}$	$\frac{}{104}$