

# YEAR 9 MATHEMATICS

## TOPIC 3

### CONSUMER ARITHMETIC [WORKSHOP]

PEN Education

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## 1 Introduction

Today we are doing many more *Consumer Arithmetic* problems. The reason for this is a) to have a little bit of a different lesson today; something a little less intense. b) To give you a chance to master your calculator (more on this as we continue). And c) to actually give you practise with some of the otherwise confusing consumer arithmetic worded problems.

Once again, here is the box of nomenclature that you should be becoming familiar with:

appreciation      GST      simple interest      compound interest      commissions      inflation      depreciation      discounts

If any of these words feel uncomfortable to you, this lesson is the time to ask your tutor what they mean!



## 2 New Problems, Old Theory

Here are some (mostly) unseen problems. If you have been attempting *ALL* of your homework carefully, then you may be able to recognise a question or two from there...

### Problems:

1. There are 740 students at a primary school, 5% of whom have red hair. Calculate the number of students in the school who have red hair. 2

**Solution:**  $740 \times 0.05 = 37$  students

2. A soccer match lasted 92 minutes (including injury time). If team A was in possession for 55% of the match, for how many minutes and seconds was team A in possession? 2

**Solution:**  $92 \times 0.55 = 50.6$  minutes, which is 50 minutes and  $0.6 \times 60 = 36$  seconds.

3. The label on a Sunnyvale tomato paste bottle says that in every 25 g serving, there are 3.6 g of carbohydrate, 0.1 g of fat, and 105mg of sodium. 4

(a) Express as a percentage of the 25 g serving:

(a) the mass of carbohydrate

**Solution:**  $\frac{3.6}{25} \times 100 = 14.4\%$

(b) the mass of fat

**Solution:**  $\frac{0.1}{25} \times 100 = 0.4\%$

(c) the mass of sodium

**Solution:** First convert sodium to grams:  $105\text{mg} = 0.105\text{g}$   
 $\frac{0.105}{25} \times 100 = 0.42\%$

- (d) The Sunnyvale website claims that the percentage of protein is 3.2 %. What mass of protein is that per 25 g serving?

**Solution:**  $25 \times \frac{3.2}{100} = 0.8$  g

4. Mt Kosciusko has a height of 2228 m, while the height of Mt Everest is 8848 m. Calculate your answers to this question correct to 3 decimal places. 2

(a) What percentage is the height of Mt Everest of the height of Mt Kosciusko?



**Solution:**  $\frac{8848}{2228} \times 100 \approx 397.129\%$

- (b) The Earth's radius is about 6400 km. What percentage of the radius of the Earth is the height of Mt Everest?

**Solution:**  $\frac{8848}{6400000} \times 100 \approx 0.138\%$

5. In the Federal Parliament, there are 150 members in the House of Representatives, of whom 37 are from Victoria.

- (a) Correct to 1 decimal place, what percentage of members are from Victoria?

**Solution:**  $\frac{37}{150} \times 100 \approx 24.7\%$

- (b) The population of Australia is about 22.6 million. What percentage of Australians are members of the House of Representatives?

**Solution:**  $\frac{150}{22600000} \times 100 \approx 0.000664\%$

6. The distance by air from Melbourne to Darwin is 3346 km, and from Melbourne to Singapore it is 6021 km. What percentage, correct to the nearest percent, is:

- (a) the Melbourne-Darwin distance of the Melbourne-Singapore distance?

**Solution:**  $\frac{3346}{6021} \times 100 \approx 55.6\%$  (rounded to 56%)

- (b) the Melbourne-Singapore distance of the Melbourne-Darwin distance?

**Solution:**  $\frac{6021}{3346} \times 100 \approx 179.9\%$  (rounded to 180%)

7. A book dealer sells rare books and charges a commission of 8% on the selling price. Find, correct to the nearest cent, the commission charged on a book that sells for these prices, and the amount that the seller eventually receives.

2

2

4



(a) \$400

**Solution:** Commission:  $400 \times 0.08 = \$32$   
Seller receives:  $400 - 32 = \$368$

**Solution:** Commission:  $575 \times 0.08 = \$46$   
Seller receives:  $575 - 46 = \$529$

(b) \$1300

**Solution:** Commission:  $1300 \times 0.08 = \$104$   
Seller receives:  $1300 - 104 = \$1196$

(d) \$142.50

**Solution:** Commission:  $142.50 \times 0.08 = \$11.40$   
Seller receives:  $142.50 - 11.40 = \$131.10$

(c) \$575

8. Shara the stockbroker charges 0.15% commission on all shares that she sells for clients. In each case, find the price at which a parcel of shares was sold if her commission was:

(a) \$30.00

**Solution:** Price of shares:  $\frac{30}{0.0015} = \$20000$

(c) \$384.75

**Solution:** Price of shares:  $\frac{384.75}{0.0015} = \$256500$

(b) \$67.35

**Solution:** Price of shares:  $\frac{67.35}{0.0015} = \$44900$

(d) \$36.51

**Solution:** Price of shares:  $\frac{36.51}{0.0015} = \$24340$

9. Madeline has received \$168000 in total simple interest payments on an investment of \$400000 that she made six years ago. What rate of interest has the bank been paying?

**Solution:** Interest rate:  $\frac{168000}{400000 \times 6} \times 100 = 7\%$

10. An investor wishes to earn \$240000 interest over a five-year period from an account that earns 12.5% p.a. simple interest. How much does the investor have to deposit into the account?

**Solution:** Principal:  $\frac{240000}{5 \times 0.125} = \$384000$

11. (a) Find, correct to 2 decimal places, the percentage decrease necessary to restore a quantity to its original value if it has been increased by:



(a) 10%

**Solution:** Decrease:  $\frac{10}{110} \times 100 \approx 9.09\%$

(c) 240%

**Solution:** Decrease:  $\frac{240}{340} \times 100 \approx 70.59\%$

(b) 22%

**Solution:** Decrease:  $\frac{22}{122} \times 100 \approx 18.03\%$

(d) 2.3%

**Solution:** Decrease:  $\frac{2.3}{102.3} \times 100 \approx 2.25\%$

(e) Find, correct to 2 decimal places, the percentage increase necessary to restore a quantity to its original value if it has been decreased by:

(a) 10%

**Solution:** Increase:  $\frac{10}{90} \times 100 \approx 11.11\%$

(b) 22%

**Solution:** Increase:  $\frac{22}{78} \times 100 \approx 28.21\%$

(c) 75%

**Solution:** Increase:  $\frac{75}{25} \times 100 = 300\%$

(d) 2.3%

**Solution:** Increase:  $\frac{2.3}{97.7} \times 100 \approx 2.35\%$

12. The radioactivity of any sample of the element strontium-90 decreases by 90.75% every century. Find the percentage reduction in radioactivity over each of the periods given below. (Calculate percentages correct to 3 decimal places.)

(a) Two centuries

**Solution:** Reduction:  $100 - (100 - 90.75)^2 \approx 99.437\%$

(b) Three centuries

**Solution:** Reduction:  $100 - (100 - 90.75)^3 \approx 99.969\%$

(c) Five centuries

**Solution:** Reduction:  $100 - (100 - 90.75)^5 \approx 99.999\%$



13. Here is a table of the annual inflation rate in Australia in the years ending 30 June 2001 to 30 June 2006 (from the Reserve Bank of Australia website).

Year Inflation rate	2001	2002	2003	2004	2005	2006
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Suppose the salary for certain jobs at Company X rises on the 1 July every year, in line with Australia's inflation rate for the financial year just past (ending 30 June).

- (a) A junior secretary earned \$40000 from 1 July 2000 to 30 June 2001. Determine how much someone in that position would earn from:

- (a) 1 July 2001 to 30 June 2002

**Solution:** Assuming an inflation rate of  $x\%$ , the salary would be:  $40000 \times (1 + \frac{x}{100})$

- (b) 1 July 2006 to 30 June 2007

**Solution:** Assuming an inflation rate of  $y\%$ , the salary would be:  $40000 \times (1 + \frac{y}{100})$

- (c) A team manager was on an annual salary of \$100000 from 1 July 2006 to 30 June 2007. Determine how much someone in that position would earn:

- (a) in the previous financial year

**Solution:** Assuming an inflation rate of  $z\%$ , the salary would be:  $100000 \div (1 + \frac{z}{100})$

- (b) from 1 July 2003 to 30 June 2004

**Solution:** Assuming an inflation rate of  $w\%$ , the salary would be:  $100000 \times (1 + \frac{w}{100})$

14. One six-year loan attracts compound interest calculated at 2%, 4%, 6%, 8%, 10% and 12% in successive years. Another six-year loan attracts compound interest calculated at 12%, 10%, 8%, 6%, 4% and 2% in successive years. Find the total percentage increase in money owing in both cases, compare the two results, and explain what has happened.

**Solution:** First loan:  $P(1.02)(1.04)(1.06)(1.08)(1.10)(1.12)$

Second loan:  $P(1.12)(1.10)(1.08)(1.06)(1.04)(1.02)$

The total percentage increase for both loans will be the same due to the commutative property of multiplication.



15. An investment at an interest rate of 10% p.a., compounded annually, returned interest of \$40000 after five years. Calculate the original amount invested.

**Solution:** Let  $P$  be the original amount.

$$P(1.10)^5 - P = 40000$$

$$P = \frac{40000}{(1.10)^5 - 1}$$

$$P \approx \$24457.88$$

16. Ms Wu's seven-year-old car is worth \$5600, and has been depreciating at 22.5% p.a. Calculate your answers to the nearest dollar.

- (a) How much was it worth four years ago?

**Solution:**  $P(1 - 0.225)^3 = 5600$

$$P \approx \$10177$$

- (b) How much was it worth seven years ago?

**Solution:**  $P(1 - 0.225)^7 = 5600$

$$P \approx \$20000$$

- (c) MsWu, however, only bought the car four years ago, at its depreciated value at that time. What has been Ms Wu's average depreciation in dollars over the four years she has owned the car?

**Solution:** Ms Wu bought the car for \$15542 and now it's worth \$5600. Depreciation over four years is  $15542 - 5600 = 9942$ . Average annual depreciation is  $\frac{9942}{4} \approx 2486$ . Ms Wu's average depreciation in dollars over the four years is approximately \$2486 per annum.

- (d) What was the average depreciation in dollars over the first three years of the car's life?

**Solution:** The value of the car after three years is  $P(0.775)^3$ .  $P(0.775)^3 = \frac{24834}{(0.775)^3}$ .  $P(0.775)^3 \approx 24834 \times 0.4420$ .  $P(0.775)^3 \approx 10976$ . Depreciation over the first three years is  $24834 - 10976 = 13858$ . Average annual depreciation is  $\frac{13858}{3} \approx 4619$ . The average depreciation in dollars over the first three years is approximately \$4619 per annum.

17. I take 900 mL of a liquid and dilute it with 100 mL of water. Then I take 900 mL of the mixture and again dilute it with 100 mL of water. I repeat this process 20 times.

- (a) What proportion of the original liquid remains in the mixture at the end?



**Solution:** Each dilution is a reduction to  $\frac{900}{1000} = 0.9$  of the previous amount. After 20 dilutions, the proportion remaining is  $0.9^{20}$ .  $0.9^{20} \approx 0.1216$ . The proportion of the original liquid remaining in the mixture is approximately 0.1216.

- (b) How much mixture should I take if I want it to contain 20 mL of the original liquid?

**Solution:** Let  $x$  be the amount of mixture to take.  $x \times 0.1216 = 20$ .  $x = \frac{20}{0.1216}$ .  $x \approx 164.47$ . Approximately 164.47 mL of the mixture should be taken to contain 20 mL of the original liquid.

18. I take a sealed glass container and remove 80% of the air. Then I remove 80% of the remaining air. I do this process six times altogether. What percentage of the original air is left in the container?

**Solution:** Each removal is keeping 20% of the previous amount. After six removals, the percentage remaining is  $(0.2)^6$ .  $(0.2)^6 = 0.000064$ .  $0.000064 \times 100 = 0.0064\%$ . The percentage of the original air left in the container is approximately 0.0064%.

2

### 3 Individual Work

Here are now 45 minutes of exact concentration that we expect you to be able to exercise. You are in Year 9 now, ideally 15 years old. You should feel accountable and comfortable to think in silence for forty-five consecutive minutes. Goodluck.

**Important:** You should really take this opportunity to become a master of your calculator. You have an expert at the front of the room. Use them!

1. Sarah decides to spend 40% of her weekly earnings on social activities, give 15% to her mother to repay a loan, and save the rest. She earns \$84 a week.

- (a) How much does Sarah spend each week on social activities?

**Solution:**  $\$84 \times 40\% = \$33.60$

- (b) What percentage of her weekly earnings does she save?

**Solution:**  $100\% - 40\% - 15\% = 45\%$

2. Nick's share portfolio consists of shares, with value \$10000, in the banking industry, \$3000 in mining shares and \$15000 in the gold market.

- (a) What percentage of his share portfolio is made up of shares in the mining sector?

2

2



**Solution:** Total value = \$10000 + \$3000 + \$15000 = \$28000 Percentage in mining =  $\frac{\$3000}{\$28000} \times 100\% = 10.71\%$

- (b) What percentage of Nick's share portfolio are not banking shares?

**Solution:** Not banking shares = \$3000 + \$15000 = \$18000 Percentage not banking =  $\frac{\$18000}{\$28000} \times 100\% = 64.29\%$

3. A real estate agent charges a commission of 8.9% on every property sale.

2

- (a) If a house sells for \$540000, how much commission will the real estate agent receive, and how much is left for the seller?

**Solution:** Commission = \$540000  $\times$  8.9% = \$48060 Seller receives = \$540000 - \$48060 = \$491940

- (b) If the real estate agent receives a commission of \$8455 for selling a house, what was the selling price of the house, and what did the seller actually receive?

**Solution:** Selling price =  $\frac{\$8455}{8.9\%} = \$95000$  Seller receives = \$95000 - \$8455 = \$86545

4. It cost the owners of the Corner Newsagency \$3500000 to run their business last year. They recorded a profit of 4.5%.

2

- (a) What was their profit last year?

**Solution:** Profit = \$3500000  $\times$  4.5% = \$157500

- (b) What was the total of their sales?

**Solution:** Sales = \$3500000 + \$157500 = \$3657500

- (c) In the previous year, their costs were \$2750000 and their sales were only \$2635000. What percentage loss did they make on their costs?

**Solution:** Loss = \$2750000 - \$2635000 = \$115000 Percentage loss =  $\frac{\$115000}{\$2750000} \times 100\% = 4.18\%$

5. Grant earned \$1260 interest on money he had invested four years ago at a simple interest rate of 4.5% p.a. How much did Grant originally invest?

2

**Solution:** Principal =  $\frac{\$1260}{4 \times 4.5\%} = \$7000$



6. A country is experiencing inflation of 12% p.a.

- (a) If the price of bread is adjusted in line with inflation, what will an annual bread bill of \$2500 become in the next year?

**Solution:** New bread bill =  $\$2500 \times (1 + 12\%) = \$2800$

- (b) If Janienne earns \$72000 in one year and \$78000 the next, is her salary increase keeping pace with inflation?

**Solution:** Salary increase =  $\$78000 - \$72000 = \$6000$  Percentage increase =  $\frac{\$6000}{\$72000} \times 100\% = 8.33\%$  No, the salary increase is not keeping pace with inflation.

7. A regional country medical centre has lost the services of one of its 16 doctors due to retirement, and has been unable to replace her.

- (a) What percentage loss is represented by the retiring doctor?

**Solution:** Percentage loss =  $\frac{1}{16} \times 100\% = 6.25\%$

- (b) If the medical centre treated 400 patients per day last year and need to reduce this number by the percentage found in part a, how many patients per day will they be able to treat in the coming year?

**Solution:** Reduced number of patients =  $400 \times (1 - 6.25\%) = 375$  patients per day

8. A shop made a profit of 6.2% on total costs last year. If the actual profit was \$156000, what were the total costs and what were the total sales?

**Solution:** Total costs =  $\frac{\$156000}{6.2\%} = \$2516129.03$  Total sales =  $\$2516129.03 + \$156000 = \$2672129.03$

9. In the January sales, the Best Dress shop has discounted all its prices by 18%.

- (a) What is the discounted price of a dress with a marked price of \$240 ?

**Solution:** Discounted price =  $\$240 \times (1 - 18\%) = \$196.80$

- (b) What was the original price of a dress with a discounted price of \$49.20 ?

**Solution:** Original price =  $\frac{\$49.20}{1 - 18\%} = \$60$



10. The number of books in a local library varies from year to year. Three years ago, the number fell by 25%, then it rose 41% the following year, and finally rose 8% last year.
- (a) What is the percentage increase or decrease over the three years, correct to the nearest 1% ?

**Solution:** Overall change =  $(1 - 25\%) \times (1 + 41\%) \times (1 + 8\%) - 1$  Overall change =  $0.75 \times 1.41 \times 1.08 - 1 \approx 0.1419$  Percentage change = 14.19% increase

- (b) If there were 429000 books in the library three years ago, approximately how many books are in the library now?

**Solution:** Current number of books =  $429000 \times 1.1419 \approx 489600$

11. The original asking price for a farm dropped by 30% a year ago, but did not attract a buyer. The price has now been further reduced by 15%.

- (a) By what percentage has the original asking price been reduced?

**Solution:** Overall reduction =  $(1 - 30\%) \times (1 - 15\%) - 1$  Overall reduction =  $0.70 \times 0.85 - 1 \approx -0.405$  Percentage reduction = 40.5%

- (b) If the farm is now for sale at \$2677500, what was the original asking price of the farm?

**Solution:** Original asking price =  $\frac{\$2677500}{1 - 40.5\%} \approx \$4500000$

12. The height of a mature tree is measured on the same day each year. Each year for the last six years, the growth has been 9% less than the previous year's growth.

- (a) What is the percentage decrease in growth over the six years, correct to the nearest percent?

**Solution:** Decrease in growth =  $1 - (1 - 9\%)^6 \approx 0.4353$  Percentage decrease = 43.53% (rounded to the nearest percent: 44%)

- (b) If the growth this year was 320 mm, what was the growth six years ago, correct to the nearest millimetre?

**Solution:** Initial growth =  $\frac{320 \text{ mm}}{(1 - 9\%)^6} \approx 563 \text{ mm}$

13. Sam's investment of \$50000 for five years earns her interest at the rate of 6.3% p.a., compounded annually.

- (a) How much will the investment be worth at the end of six years?



**Solution:** Future value =  $\$50000 \times (1 + 6.3\%)^6 \approx \$70968.77$

- (b) What is the percentage increase of her original investment at the end of six years?

**Solution:** Percentage increase =  $\frac{\$70968.77 - \$50000}{\$50000} \times 100\% \approx 41.94\%$

- (c) What is the total interest earned over the six years?

**Solution:** Total interest =  $\$70968.77 - \$50000 = \$20968.77$

- (d) What would the simple interest on the investment have been, assuming the same interest rate of 6.3% p.a.?

**Solution:** Simple interest =  $\$50000 \times 6.3\% \times 6 = \$18900$

14. A company buys new company cars every three years. At the end of the three years, it offers them for sale to the employees on the assumption that they have depreciated at 30% p.a. The company is presently advertising some cars at \$30000 each.

- (a) What did each car cost the company originally, correct to the nearest thousand dollars?

**Solution:** Original cost =  $\frac{\$30000}{(1-30\%)^3} \approx \$78100$

- (b) What is the average depreciation in dollars p.a., correct to the nearest hundred dollars, on each car over the three-year period?

**Solution:** Average depreciation p.a. =  $\frac{\$78100 - \$30000}{3} \approx \$16033.33$

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## 4 Group Race

This could either be really fun or really chaotic. Either way, here are 8 challenging consumer arithmetic questions for you all to chew on.

1. The population of a town decreases by 15% during 2010. What percentage increase, correct to 2 decimal places, is necessary during 2011 for the population to be restored to the level it was at immediately before the decrease in 2010?

2



**Solution:** Let the original population be  $P$ . After a decrease of 15%, the new population is  $P \times (1 - 0.15) = P \times 0.85$ . To restore the population to  $P$ , we need to solve for the percentage increase  $x$  such that:

$$P \times 0.85 \times \left(1 + \frac{x}{100}\right) = P$$

Simplifying, we get:

$$0.85 \times \left(1 + \frac{x}{100}\right) = 1$$

$$1 + \frac{x}{100} = \frac{1}{0.85}$$

$$\frac{x}{100} = \frac{1}{0.85} - 1$$

$$x = \left(\frac{1}{0.85} - 1\right) \times 100$$

$$x = (1.17647 - 1) \times 100$$

$$x = 0.17647 \times 100$$

$$x = 17.647$$

The necessary percentage increase is 17.65% (rounded to two decimal places).

2. The length of a rectangle is increased by 15% and the width is decreased by 11%. What is the exact percentage change in the area?

2

**Solution:** Let the original length be  $L$  and the original width be  $W$ . The original area is  $A = L \times W$ . The new length is  $L \times 1.15$  and the new width is  $W \times 0.89$ . The new area is:

$$A' = (L \times 1.15) \times (W \times 0.89)$$

$$A' = L \times W \times 1.15 \times 0.89$$

$$A' = A \times 1.15 \times 0.89$$

The percentage change in the area is:

$$\frac{A' - A}{A} \times 100\%$$

$$= (1.15 \times 0.89 - 1) \times 100\%$$

$$= (1.0235 - 1) \times 100\%$$

$$= 0.0235 \times 100\%$$

$$= 2.35\%$$

The exact percentage change in the area is 2.35%.



3. The radius of a circular pool of water increases by 8%. What is the exact percentage change in the area of the pool of water?

**Solution:** Let the original radius be  $r$ . The area of the circle is  $A = \pi r^2$ . The new radius is  $r' = r \times 1.08$ . The new area is:

$$A' = \pi(r')^2$$

$$A' = \pi(r \times 1.08)^2$$

$$A' = \pi r^2 \times 1.08^2$$

$$A' = A \times 1.08^2$$

The percentage change in the area is:

$$\begin{aligned} & \frac{A' - A}{A} \times 100\% \\ &= (1.08^2 - 1) \times 100\% \\ &= (1.1664 - 1) \times 100\% \\ &= 0.1664 \times 100\% \\ &= 16.64\% \end{aligned}$$

The exact percentage change in the area of the pool of water is 16.64%.

4. The area of a circular pool of oil is increased by 8%. What is the percentage increase of the radius?



**Solution:** Let the original area be  $A$  and the original radius be  $r$ . The new area is  $A' = A \times 1.08$ . Since the area of a circle is  $A = \pi r^2$ , we have:

$$\begin{aligned} A' &= \pi(r')^2 \\ A \times 1.08 &= \pi(r')^2 \\ \frac{A \times 1.08}{\pi} &= (r')^2 \\ r' &= \sqrt{\frac{A \times 1.08}{\pi}} \\ r' &= \sqrt{\frac{\pi r^2 \times 1.08}{\pi}} \\ r' &= r \times \sqrt{1.08} \end{aligned}$$

The percentage increase of the radius is:

$$\begin{aligned} &\frac{r' - r}{r} \times 100\% \\ &= (\sqrt{1.08} - 1) \times 100\% \\ &\approx (1.03923 - 1) \times 100\% \\ &\approx 0.03923 \times 100\% \\ &\approx 3.923\% \end{aligned}$$

The percentage increase of the radius is approximately 3.92% (rounded to two decimal places).

5. A man earns a salary of \$1440 for a 44-hour week. His weekly salary is increased by 10% and his hours are reduced by 10%. Find his new hourly salary. 2

**Solution:** The original hourly salary is  $\frac{\$1440}{44 \text{ hours}} = \$32.7272$  per hour. The new weekly salary is  $\$1440 \times 1.10 = \$1584$ . The new number of hours is  $44 \times 0.90 = 39.6$  hours. The new hourly salary is:

$$\frac{\$1584}{39.6 \text{ hours}} \approx \$40$$

per hour.

6. In a particular country in 2010, 15% of the population is unemployed and 85% is employed. In 2011, 10% of the people unemployed became employed and 10% of those employed became unemployed. What percentage of the population is employed now? 2



**Solution:** Let the total population be  $P$ . Initially,  $0.85P$  is employed and  $0.15P$  is unemployed. In 2011,  $0.10 \times 0.15P = 0.015P$  of the unemployed become employed, and  $0.10 \times 0.85P = 0.085P$  of the employed become unemployed. The new number of employed people is:

$$\begin{aligned} &0.85P - 0.085P + 0.015P \\ &= 0.85P - 0.07P \\ &= 0.78P \end{aligned}$$

The percentage of the population that is employed now is 78%.

7. The number of trees on Green Plateau fell by 5% every year for 10 years. Then the numbers rose by 5% every year for 20 years. What was the total percentage gain or loss of trees over the 30-year period?

**Solution:** Let the original number of trees be  $T$ . After 10 years of decreasing by 5%, the number of trees is:

$$T \times (1 - 0.05)^{10}$$

After 20 more years of increasing by 5%, the number of trees is:

$$T \times (1 - 0.05)^{10} \times (1 + 0.05)^{20}$$

The total percentage gain or loss is:

$$\begin{aligned} &\frac{T \times (1 - 0.05)^{10} \times (1 + 0.05)^{20} - T}{T} \times 100\% \\ &= ((1 - 0.05)^{10} \times (1 + 0.05)^{20} - 1) \times 100\% \\ &= ((0.95)^{10} \times (1.05)^{20} - 1) \times 100\% \\ &\approx (0.59874 \times 2.65330 - 1) \times 100\% \\ &\approx (1.58846 - 1) \times 100\% \\ &\approx 0.58846 \times 100\% \\ &\approx 58.846\% \end{aligned}$$

The total percentage gain of trees over the 30-year period is approximately 58.85% (rounded to two decimal places).

8. Particular shares were released in the stock market and lost, per day, an average of 2.23% of their original value over the first four days. Over the first day, the shares increased in value by 15%, and over the second day, a further 10% increase was recorded. However, a 20% decrease in the share value occurred on the third day. What percentage decrease was recorded over the fourth day?



**Solution:** Let the original value of the shares be  $S$ . After the first day, the value is  $S \times 1.15$ . After the second day, the value is  $S \times 1.15 \times 1.10$ . After the third day, the value is  $S \times 1.15 \times 1.10 \times 0.80$ . The total loss over four days is  $2.23\% \times 4 = 8.92\%$  of the original value. The value after four days is  $S \times (1 - 0.0892)$ . We have:

$$S \times 1.15 \times 1.10 \times 0.80 \times (1 - x) = S \times (1 - 0.0892)$$

$$1.15 \times 1.10 \times 0.80 \times (1 - x) = 1 - 0.0892$$

$$0.92 \times (1 - x) = 0.9108$$

$$1 - x = \frac{0.9108}{0.92}$$

$$1 - x = 0.99043$$

$$x = 0.00957$$

$$x = 0.957\%$$

The percentage decrease recorded over the fourth day is 0.957%.

## 5 Homework

There is no homework for this week :D