Proof. Let $\varepsilon > 0$. Since $x_n \to x$ and $(x) \geq 0$, there exists $N_1 \in \mathbb{N}$ such that $n \geq N_1$ implies $|x_n - x| < \frac{\varepsilon}{2}$. By the reverse triangle inequality, this gives $|x_n| \geq |x| - |x_n - x| > \frac{\varepsilon}{2}$, so $|x_n| > \frac{\varepsilon}{2}$ for all $n \geq N_1$. Since $x \neq 0$, choose $N_2 \in \mathbb{N}$ such that $n \geq N_2$ implies $|x_n - x| < \frac{\varepsilon}{2}$. Let $N = \max(N_1, N_2)$. Then for $n \geq N$,

$$\begin{aligned} \left|\frac{1}{x_n} - \frac{1}{x}\right| &= \frac{|x - x_n|}{|x_n||x|} \\ &< \frac{\frac{\varepsilon}{2}|x|}{\frac{\varepsilon}{2}|x|} = \varepsilon. \end{aligned}$$

∎