Proof. Let $\varepsilon > 0$. Since $\lim_{n\to\infty} x_n = x$, there exists $N_1 \in \mathbb{N}$ such that $n \geq N_1$ implies $|x_n - x| < \frac{\varepsilon}{2}$. Since $\lim_{n\to\infty} y_n = y$, there exists $N_2 \in \mathbb{N}$ such that $n \geq N_2$ implies $|y_n - y| < \frac{\varepsilon}{2}$. Let $N = \max\{N_1, N_2\}$. Then for $n \geq N$, for the triangle inequality,

$$|(x_n + y_n) - (x + y)| \leq |x_n - x| + |y_n - y| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

∎