Proof. Let A be the set of all nx, where n runs through the positive integers. If ?? were false ($nx <= y$), then y would be an upper-bound of A. But then A has a least upper bound in R. Put $\alpha = \sup A$. Since $x > 0$, $\alpha - x < \alpha$, and $\alpha - x$ is not an upper bound in R. Hence $\alpha - x < mx$ for some positive integer m. But then $\alpha < (m+1)x \in A$, which is impossible, since α is an upper bound of A.

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