Proposition 0.0.1. *The unique solution of the SDE*

$$dX_t = \mu(t, X_t)\,dt + \sigma(t, X_t)\cdot d\boldsymbol{W}_t$$

with

$$\begin{aligned}\mu(t, X_t) &= A_t X_t + a_t,\\ \sigma(t, X_t) &= [B_t^1 X_t + b_t^1, \cdots, B_t^d X_t + b_t^d],\end{aligned}$$

where $A,\ a,\ B^i,\ b^i\ (i = 1, \cdots, d)$ *are all* $\mathcal{F}$*-adapted bounded processes, is given by the formula*

$$X_t = \Phi_t\left(X_0 + \int_0^t \Phi_s^{-1}[a_s - \boldsymbol{B}_s \cdot \boldsymbol{b}_s]\,ds + \int_0^t \Phi_s^{-1}\boldsymbol{b}_s \cdot d\boldsymbol{W}_s\right)$$

where

$$\Phi_t = \exp\left(\int_0^t \left(A_s - \frac{\boldsymbol{B}_s \cdot \boldsymbol{B}_s}{2}\right)ds + \int_0^t \boldsymbol{B}_s \cdot d\boldsymbol{W}_s\right).$$