Proof. Let $\frac{1}{\ln 2} > 0$ and $n \in \mathbb{N} > \log_{\frac{1}{2}} 2$ the Archimedean Property of Natural Numbers. Then

$$\begin{aligned} n &> \log_{\frac{1}{2}} 2 \\ &= \frac{\log 2}{\log \frac{1}{2}} \\ &= \frac{\log 2}{\log 2^{-1}} \\ &= \frac{1}{-1} \cdot \frac{\log 2}{\log 2} \\ &= -1 \cdot \frac{\log 2}{\log 2} \\ \Rightarrow \log 2^n &= \log 2 \cdot n \\ &= 2^n \cdot \log 2 \\ &= 2^n \cdot 1 \cdot \log \end{aligned}$$

Which proves the positive case of our absolute value definition of the limit. Note that the negative case is impossible for natural numbers n. ∎