Proof. (i) $\Omega \in \mathcal{F}$ by assumption.

(ii) Closure under complement: Let $A \in \mathcal{F}$. Since $\Omega \in \mathcal{F}$ and $\mathcal{F}$ is closed under set difference,

$$A^c = \Omega \setminus A \in \mathcal{F}.$$

(iii) Closure under finite unions: Let $A, B \in \mathcal{F}$. Then $B^c \in \mathcal{F}$ by (ii), so

$$A \setminus B^c \in \mathcal{F}$$

by closure under set difference. But

$$A \setminus B^c = A \cap B.$$

Hence $A \cap B \in \mathcal{F}$.

Now using complements again,

$$A \cup B = (A^c \cap B^c)^c,$$

and since $A^c, B^c \in \mathcal{F}$ and $\mathcal{F}$ is closed under intersection (as just shown), we get

$$A \cup B \in \mathcal{F}.$$

■