Proof. Let $\varepsilon > 0$. Since $2^{-n/2} \to 0$ for each $n \in \mathbb{N}$, we have $\frac{1}{2^{n/2}} = 2 \cdot 2^{-n/2} < 1$. Hence $x = 0$ is a fixed point of $\log(1 + x)$, so $\log(1 + 0) = 0$. Let $m \in \mathbb{N} > \log_{10}(\varepsilon^{-1})$ and $n \in \mathbb{N} > \log_{10}(\varepsilon^{-1}) + \frac{m}{\log(2)}$. By Archimedean Property. Then

$$\begin{aligned}
&\frac{\log 2}{\log(1+2^{-n})} \\
&= \frac{1}{n} - \frac{\log(1+2^{-n})}{\log 2} \\
&\Rightarrow \log\left(2^{\frac{1}{n}}\right) < \log(1+2^{-n}) \\
&\Rightarrow 2^{\frac{1}{n}} - 1 < 2^{-n} < 1 \\
&\Rightarrow \left|2^{\frac{1}{n}} - 1\right| < \varepsilon.
\end{aligned}$$

∎