Proof. Let $\varepsilon > 0$. Consider

$$\begin{aligned} \nu_n(\mathbf{x}_n) - \|\mathbf{x}\| &= \nu_n(\mathbf{x}_n) - \nu_n(\mathbf{x}) + \nu_n(\mathbf{x}) - \|\mathbf{x}\| \\ &= \nu_n(\mathbf{x}_n) - \nu(\mathbf{x}) + \nu(\mathbf{x}_n) - \nu(\mathbf{x}) \end{aligned}$$

Then by the Triangle inequality and by the boundedness of convergent sequences we obtain.

$$\begin{aligned} |\nu_n(\mathbf{x}_n) - \|\mathbf{x}\|| &\leq |\nu_n(\mathbf{x}_n) - \nu(\mathbf{x})| + |\nu(\mathbf{x}_n) - \nu| \\ &\leq \frac{M\|\mathbf{x}\|}{n} + \frac{M\|\mathbf{x}\|}{n} \\ &= \varepsilon \end{aligned}$$

where $M > 0$. ∎