Proof. Let A be the set of all x such that x is a positive integer. If A were finite (say $\{x, y\}$), there is would be an upper bound of A. But then, A has a least upper bound in $\mathbb{N}$. Put $x = \sup A$. Since $x \in \mathbb{N}$, $x + 1$ is also in A, so x is not an upper bound of A. Hence $x + 1 > x$ for some positive integer x. But then $x + 1 > x$, $x \in A$, which is impossible, since x is an upper bound of A. $\square$