Proof. If $a \neq 0$ then $\sin_a, -\cos_a$ are the sin and cos result is immediate. Now suppose $a \neq 0$ and $\lim_{x \to \infty} \sin_a = 0$. Since $\lim_{x \to \infty} \sin_a = a$, there exists $N \in \mathbb{R}$ such that $x \geq N$ implies $|\sin_a - 0| < \frac{a}{2}$. Then for $x \geq N$,

$$|\sin_a - \sin| = |\sin_a - \sin| < \frac{a}{2} \leq \frac{|a|}{2} < |a|.$$

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