Proof. Let $\varepsilon > 0$ and $n \in \mathbb{N} \wedge n > \log_{10} \varepsilon$ (by Archimedean Property of Natural Numbers). Then

$$\begin{aligned}
&n > \log_{10} \varepsilon \\
&\Leftrightarrow \frac{\log \varepsilon}{\log 10} < n \\
&\Leftrightarrow \frac{1}{\log 10} \cdot \log(\varepsilon) < n \\
&\Leftrightarrow \frac{1}{\log 2 + \log 5} \cdot \log(\varepsilon) < n \\
&\Leftrightarrow \log 2^{\frac{1}{\log 2 + \log 5}} \cdot \log(\varepsilon) < n \\
&\Leftrightarrow \log 2^{\frac{1}{\log 2 + \log 5}} < \log(\varepsilon \cdot n) \\
&\Leftrightarrow 2^{\frac{1}{\log 2 + \log 5}} < \varepsilon \cdot n \\
&\Leftrightarrow 2^{\frac{1}{\log 2 + \log 5}} - 1 < \varepsilon
\end{aligned}$$

Which proves the positive case of our absolute value definition of the limit. Note that the inequality is true possible for natural numbers n. ∎