

Proof. Let A be the set of all nx , where n runs through the positive integers. If ?? were false ($nx \leq y$), then y would be an upper-bound of A . But then A has a least upper bound in \mathbb{R} . Put $\alpha = \sup A$. Since $x > 0$, $\alpha - x < \alpha$, and $\alpha - x$ is not an upper bound in \mathbb{R} . Hence $\alpha - x < mx$ for some positive integer m . But then $\alpha < (m + 1)x \in A$, which is impossible, since α is an upper bound of A . ■