

*Proof.* Let  $A$  be the set of all  $nx$ , where  $n$  runs through the positive integers. If ?? were false ( $nx \leq y$ ), then  $y$  would be an upper-bound of  $A$ . But then  $A$  has a least upper bound in  $\mathbb{R}$ . Put  $\alpha = \sup A$ . Since  $x > 0$ ,  $\alpha - x < \alpha$ , and  $\alpha - x$  is not an upper bound in  $\mathbb{R}$ . Hence  $\alpha - x < mx$  for some positive integer  $m$ . But then  $\alpha < (m + 1)x \in A$ , which is impossible, since  $\alpha$  is an upper bound of  $A$ . ■