Proof. Let $\epsilon > 0$ and $n \in \mathbb{N} \wedge n > \log_{10} \epsilon$ (by Archimedean Property of Natural Numbers). Then

$$\begin{aligned}
&n > \log_{10} \epsilon \\
&\Leftrightarrow \frac{\log \epsilon}{\log 10} < n \\
&\Leftrightarrow \frac{1}{\log 10} \cdot \log(\epsilon) < n \\
&\Leftrightarrow \frac{1}{\log 2} \cdot \log(\epsilon) < n \\
&\Leftrightarrow \log 2^n < \log(\epsilon + 1) \\
&\Leftrightarrow 2^n < \epsilon + 1 \\
&\Leftrightarrow 2^n - 1 < \epsilon
\end{aligned}$$

Which proves the positive case of our absolute value definition of the limit. Note that the inequality was possible for natural numbers. ∎